UNIT 1

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Syllabus

- **Introduction:** Demand of Information Age, Block Diagram of Optical fiber Communication System, Technology used in OFC System,
- **Structure and types of Fiber:** modes and Configuration, mode theory for circular guide modal equation, modes in optical fiber, linearly polarized modes, attenuation factors,
- **Pulse Broadening in Optical Fiber:** single mode fiber, mode field diameter, signal distortion in single mode fiber, Derivation of material dispersion and waveguide dispersion. Attenuation, Signal Degradation in Optical Waveguides, Pulse Broadening in Graded index fiber Waveguides, Mode Coupling.

Lecture Plan

UNIT-I INTRODUCTION		
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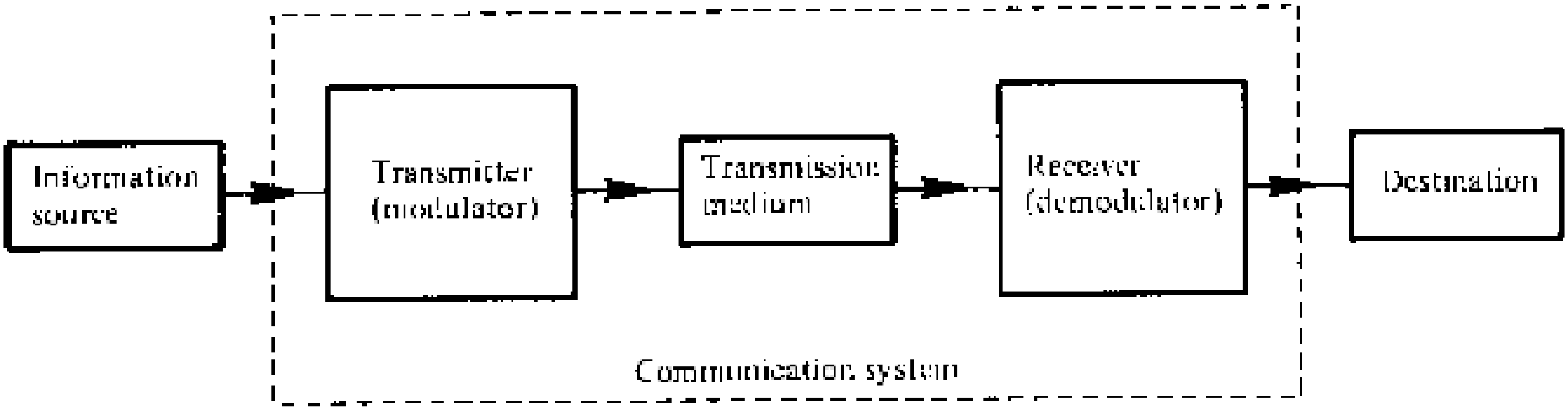
Introduction [1]

- Communication is basically the transfer of information from one point to another point, i.e., from source to destination.
- So, in order to convey any information over any distance we require communication system.
- Within a communication system the information transfer is achieved by superimposing the information on to the carrier.

Cont…

- The carrier is usually an electromagnetic wave.
- This process of superimposing the information on to the carrier is called modulation.

Block Diagram of Communication Sys



Cont…

- The function of communication system is to convey information from source to destination.
- The information is conveyed over a distance, i.e. Medium or channel.
- In case of electrical communication system, the information source provides electrical signal.
- The transmitter consists of electrical/electronic components.

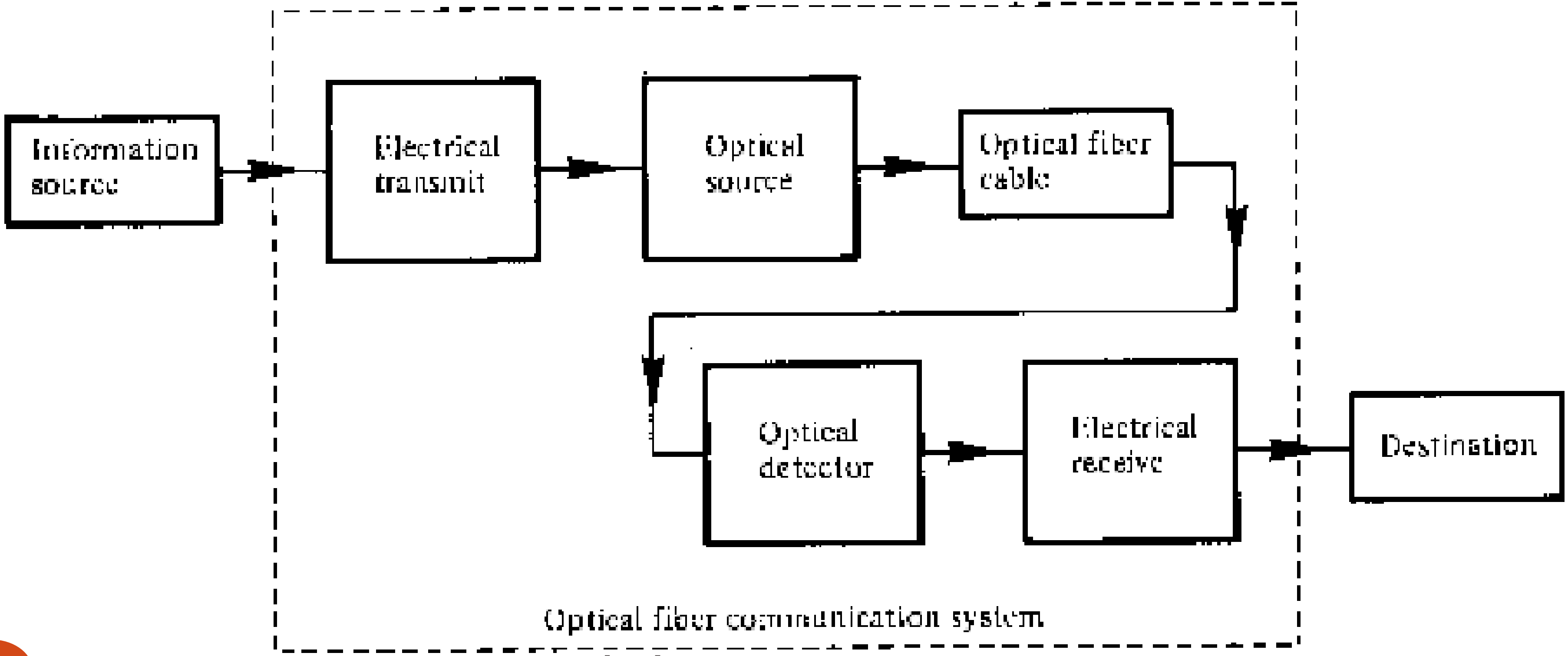
Cont...

- The role of transmitter is to convert the information signal into a suitable form, that the medium can handle.
- The information signal is converted by modulating the carrier.
- The medium or channel is usually the pair of wires or a radio link.
- During any kind of transmission the signal is attenuation i.e. It suffers losses.

Cont…

- Additionally distortions may occur due to some unpredictable mechanisms within the system.
- So it means that any communication system is distance limited.
- For efficient transmission the channel length must not exceed the allowable distance.
- For long distance communication, we need to install repeaters or line amplifiers.

Block Diagram of OFC System [1]



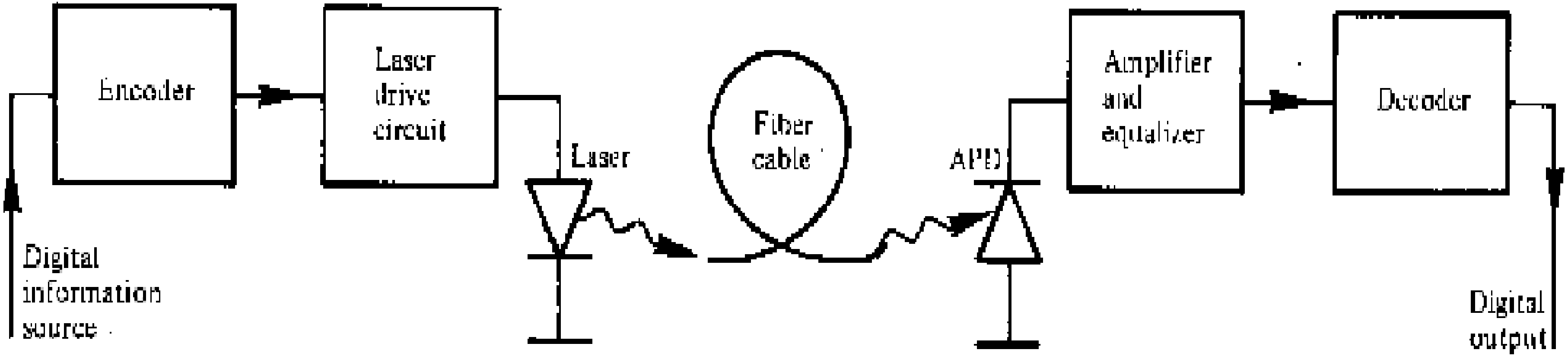
Cont…

- In OFC system, the information source provides an electrical signal & is derived by a signal which is not electrical (e.g. sound).
- The information is fed into the electrical transmit block.
- The electrical stage drives an optical source to give modulation of light-wave carrier.
- The optical source provides the electrical – optical conversion. Optical source is usually LASER or LED.

Cont...

- The medium in case of OFC system is optical fiber cable.
- After that we have optical detector, which may be either photodiode, photoconductor or phototransistor.
- The function of optical detector here is to convert optical light-wave into electrical signal.
- So, at either end of the link there is a requirement of electrical interfacing.
- The signal processing is usually performed electrically.

Digital OFC Link [1]



Cont…

- An optical carrier may be modulated by either analog or digital information signal.
- In case of digital modulation discrete changes in the light intensity takes place (on – off pulses).
- Analog links are simple to implement, less efficient and requires high value of SNR than Digital links.
- So, the analog OFC links are not preferred over digital OFC links.

Cont…

- In digital OFC links the input digital signal is encoded for optical transmission.
- LASER drive circuit modulates the intensity of LASER with the encoded digital signal.
- After getting digital optical signal from LASER it is launched into the optical fiber cable.
- In order to receive this signal we have APD.

Cont…

- Amplifier and equalizer provides gain and reduces noise bandwidth.
- In order to obtain original digital signal, the amplified signal is decoded.

Advantages of OFC [1], [2]

1. Small size and light weight:

- optical fiber have very small diameter, less than the diameter of human hair., even the fibers are covered by protecting coating.
- They are much lighter than copper cables.
- It makes them useful in aircraft, satellites and ships, where light weight cables are advantageous.

Cont…

2. <u>Immune to electrical interference:</u>

- Since the optical fiber cable is a dielectric material, so it doesn't conduct electricity.
- This makes optical fiber immune to the electromagnetic interference (EMI).
- So the OFC system remains unaffected by transmission through an electrically noisy environment.

Cont…

3. Low transmission loss & long distance transmission:
   - Optical fibers are fabricated with loss as low as 0.2 dB/km.
   - So they exhibit very low attenuation and transmission losses.
   - So the repeaters are required for very long distance transmission.
   - It makes OFC link cheaper and simpler.
   - This makes them useful for long distance transmission.

Cont…

4. <u>Increased signal security:</u>
   - Since the optical fiber is well confined waveguide, and there is an opaque coating around the fiber which absorbs any signal emissions.
   - Therefore they provide high degree of signal security.
   - This makes them useful in information security applications, such as financial, legal, government, and military systems.

Cont…

5. <u>Safety:</u>
   - Since optical fiber doesn't have grounding, sparks and high voltage problems, that's why they are more secure than copper cables.
   - However LASER emissions may damage eyes.
6. <u>Flexibility:</u>
   - Fibers may bent to some small radii or twisted without damage. They provides high degree of flexibility.

Basic Optical Laws [1] [2]

a. Refractive Index
b. Total Internal Reflection (TIR)
c. Numerical Aperture & Acceptance Angle

a. Refractive Index

- The fundamental optical parameter of the material is refractive index or simply index of refraction.
- Speed of light in free space is $3X10^8$ m/s.
- It is related to frequency and light as $c = f\lambda$.
- After entering a dielectric or non-conducting medium the wave travels at a different velocity v which is less than c.
- The ratio of the speed of light in a vacuum to that in any medium is known as refractive index n.

Cont…

- Refractive index is given by: $n = \frac{c}{v}$
- For air n is 1.00, for water 1.33, for silica glass 1.45, and for diamond it is 2.42.

b. Total Internal Reflection (TIR)

- Consider 2 different medium with refractive indices n_1 & n_2 respectively.
- When a ray encounters a boundary separating two different materials, some part of the ray is reflected back into the first medium and remaining is bent or refracted into second.
- This is because of the difference in speed of the light in the different medium.

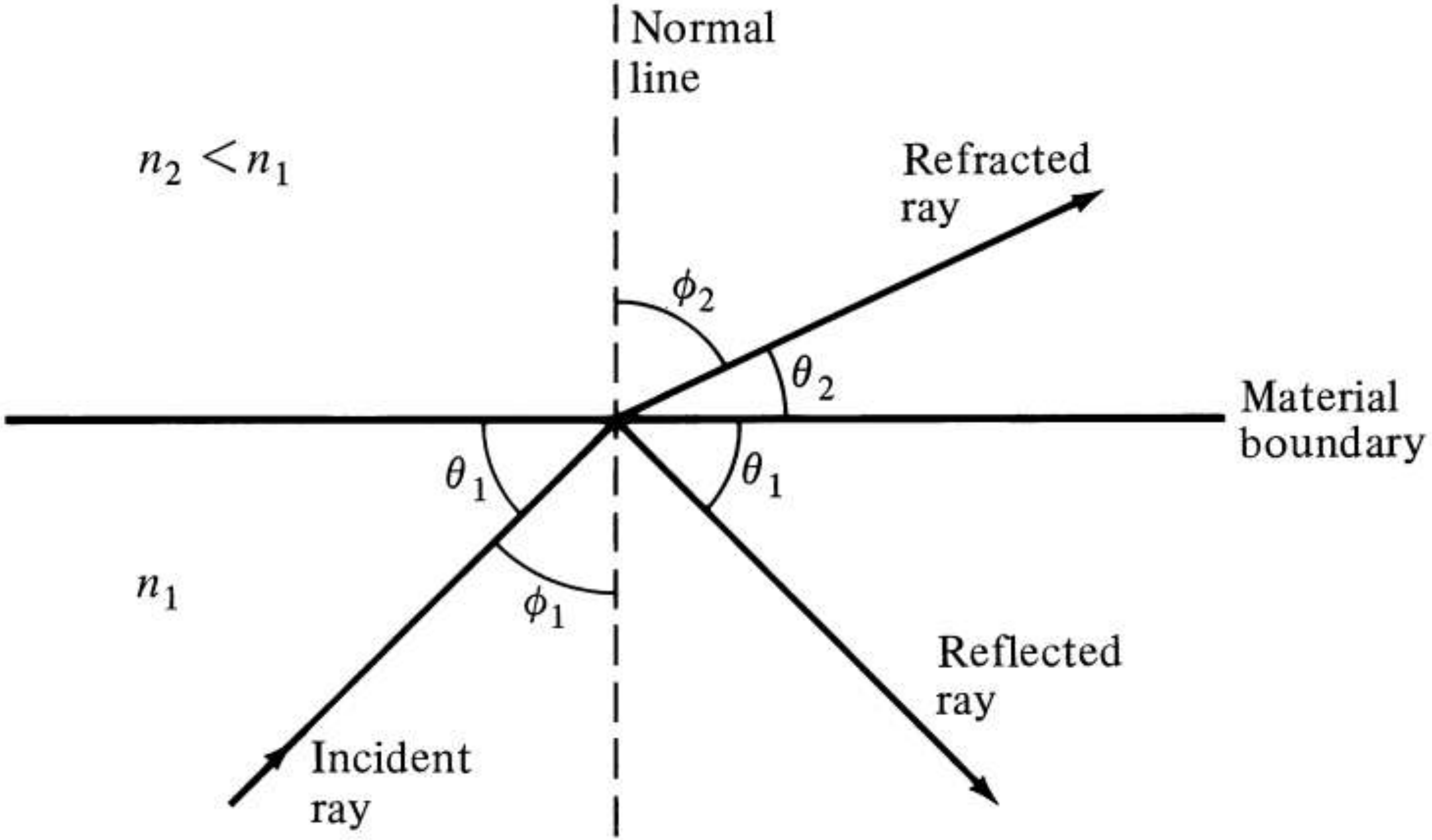


Fig.: Refraction and reflection

Cont...

- Let ϕ_1 is the angle of incidence & is measured between the incident ray and normal to the surface.
- Let ϕ_2 is the angle of refraction & is measured between the refraction ray and normal to the surface.
- Then according to the Snell's law

$$n_1 \sin \phi_1 = n_2 \sin \phi_2$$

- When ϕ_1 becomes larger, ϕ_2 approaches 90^0.
- At this point, no refraction is possible.

Cond...

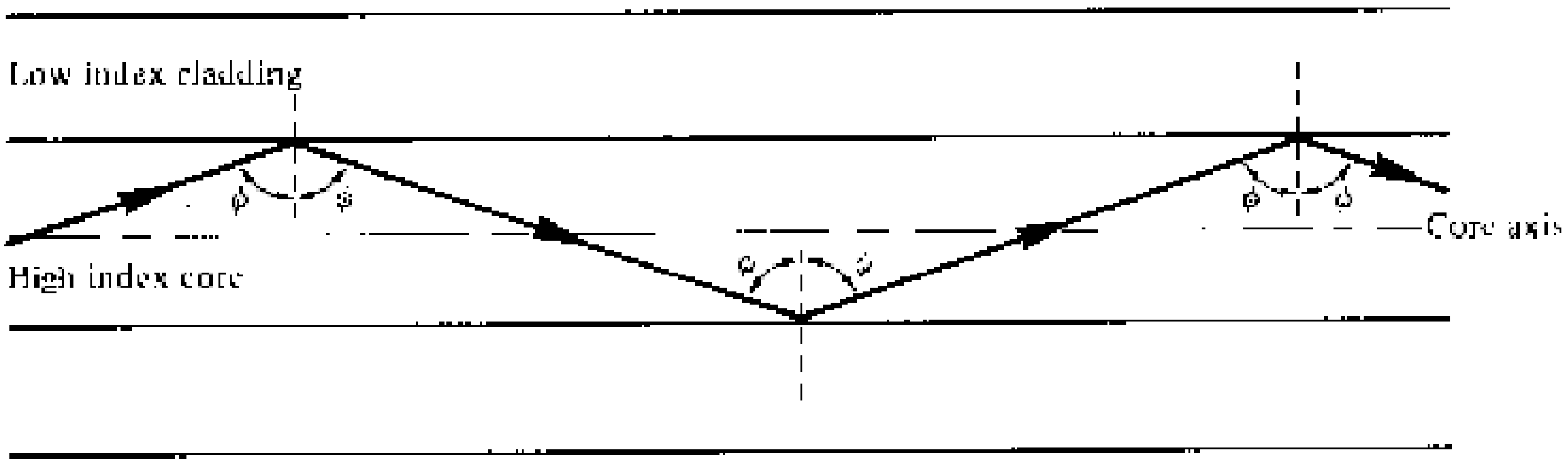
- In this case light ray becomes "*total internally reflected*."
- The conditions required for total internal reflection can be determined by using Snell's Law.
- So for total internal reflection .
- At this condition angle of incidence is known as critical angle $\phi_2 = 90^0$
- So, the critical angle can be calculated as,

ϕ_c

$$\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Cond...

- The light ray shown on next slide is known as meridional ray since this kind of ray passes through the axis of the fiber core.
- This kind of optical fiber is assumed to be perfect in nature.



The transmission of a light ray in a perfect optical fiber.

c. Numerical Aperture & Acceptance Angle

- From Snell's law the minimum angle that supports total internal reflection for the meridional ray is given by:

$$\sin\phi_{\min} = \frac{n_2}{n_1}$$

- If the rays strikes the core-cladding interface at n angle less than this angle will be refracted out of the core and will be lost in cladding.
- Now, consider the right angled triangle on next slide.
- we can say

$$n\sin\theta_0 = n_1\sin\theta$$

Cond...

- But, $\phi = \frac{\pi}{2} - \theta$ hence $n \sin\theta_0 = n_1 \cos\phi = n_1 (1 - \sin^2\phi)^{1/2}$
- if $\phi = \phi_c$ then $\theta_0 = \theta_{0,\max}$ it is acceptance angle for which TIR takes place
- Then, $n \sin\theta_{0,\max} = (n_1^2 - n_2^2)^{1/2}$. The quantity on left side of this equation is known as NA (Numerical Aperture).
- So, numerical Aperture is given by:

$$NA = n \sin\theta_{0,\max} = (n_1^2 - n_2^2)^{1/2} \approx n_1 \sqrt{2\Delta}$$

- The parameter Δ is known as core-cladding index difference or simply index difference given by $\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \approx 1 - \frac{n_2}{n_1}$

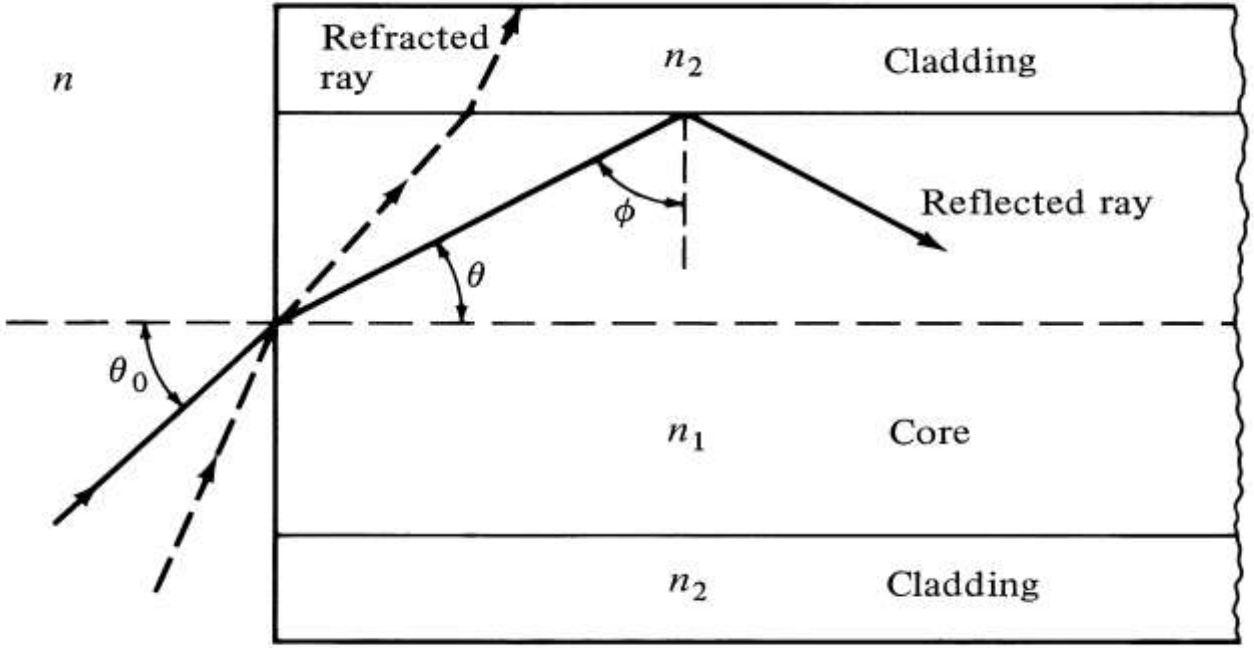


Fig. Meridional ray representation

Optical Fiber modes and configurations [1] [2]

a. Fiber types
b. Rays and modes
c. Step-index fiber structure
d. Ray optic representation

a. Fiber Types

- An optical fiber is a dielectric wave guide.
- It confines electromagnetic energy in the form of light within the surface of the waveguide.
- It guides the light in a direction parallel to it axis.
- The propagation of light along a waveguide can be described in terms of a guided electromagnetic waves.
- These waves are known as "*modes*" of the waveguide.

Cond…

- Each guided mode is pattern of electric and magnetic field distributions.
- This pattern is repeated along the fiber at equal intervals.
- Only certain no. of modes are capable of propagating along the waveguide.
- These modes are electromagnetic waves and satisfies homogenous wave equation in the fiber and the boundary condition at the waveguide surface.

Cond...

- The most widely configuration is a single solid dielectric cylinder of radius and refractive index n_1.
- This cylinder is known as the core of the fiber.
- The core is surrounded by a solid dielectric material known as cladding with a refractive index n_2 ($< n_1$)
- Cladding is not for propagating light.

Cond...

- The purposes of the core are:-
  - To reduce scattering losses.
  - It provides mechanical strength.
  - It protects core from absorbing surface contaminants.
- The core of the standard optical fiber is made up of pure silica glass (SiO_2).
- The core is surrounded by a silica glass cladding.
- The fiber is encapsulated in an elastic plastic material.

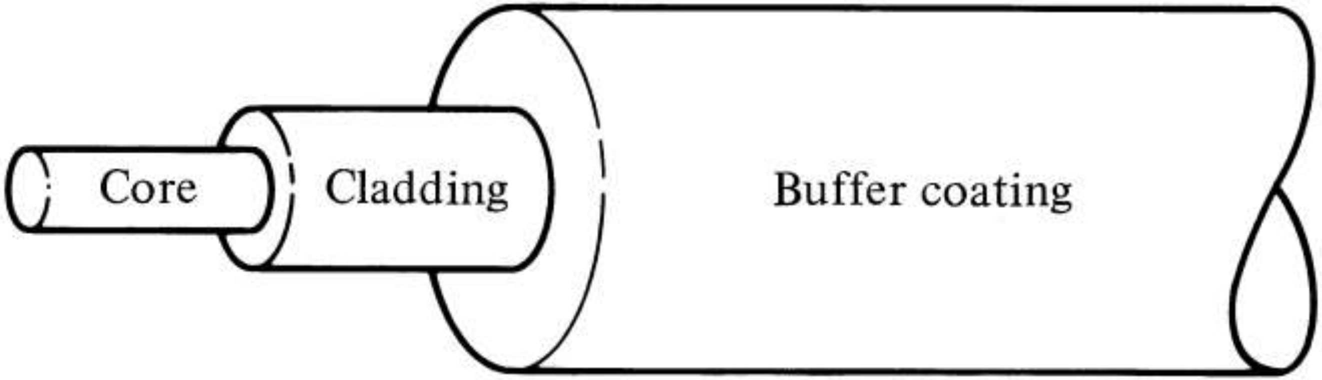


Fig. Single fiber structure

Cond…

- This plastic material is known as buffer coating.
- This material provides additional strength to the fiber.
- This also isolates the fibers from distortions or roughness of the adjacent surface.
- On the bases of the variations in the core material, we have two kind of optical fibers as shown in figure on next slide.

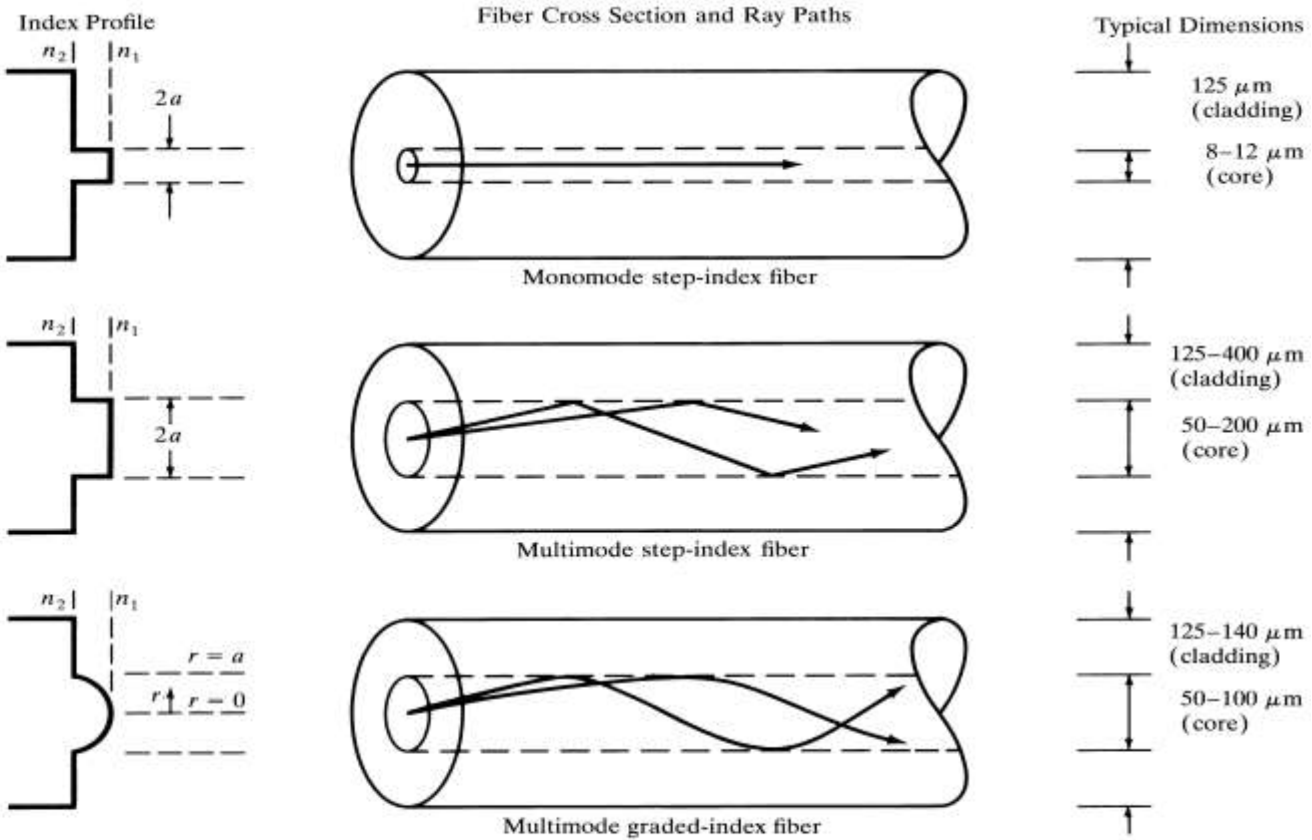


Fig. : Comparison of fiber structures

Cond...

- If the refractive index of the core is uniform throughout and if there is an abrupt change (step) at the cladding boundary, then this is called **step index fiber**.
- If the refractive index of the core varies as a function of the radial distance (gradual) from the centre of the fiber, this is called **graded index fiber**.
- Both the step index and graded index fibers can be further classified into single mode and multi mode fibers.

Cond…

- The single mode fibers contain only one mode of propagation, whereas multimode fibers contain many hundred modes.

Single Mode Vs Multi Mode Fibers

- Since the multimode fibers have larger core radii, it is easy to launch optical fiber into multimode mode fiber.
- Optical power can be launched into multimode fiber using LED, whereas single-mode fibers are generally excited using lasers.
- The multimode fibers suffer from intermodal dispersion.

b. Rays and modes

- We know that the light is an electromagnetic wave.
- It is to be guided along the axis of the optical fiber.
- It is represented by the superposition of bounded or trapped modes.
- Also note that the guided modes consists of a set of simple electromagnetic field configurations.

Cond...

- For monochromatic light fields there is an exponential factor & is given by: $e^{j(\omega t-\beta z)}$
- Where,

$$\omega \text{ is frequency in radian}$$
$$z \text{ is the direction } (+\text{ve}) \text{ of propagation of light}$$
$$\beta \text{ is the z component of the wave propagation constant } k$$
$$k = \frac{2\pi}{\lambda}$$
$$\lambda \text{ is known as wavelength}$$

- The mode field can be determined using Maxwell's equation and boundary conditions at core-cladding interface.

c. Step-index fiber structure

- In practical step-index fiber the core of radius a has a refractive index n_1 (1.48).
- Let this is surrounded by a cladding of radius n_2, where $n_2 = n_1(1-\Delta)$
- The parameter Δ is called core-cladding index difference or simple index difference.
- Since $n_2<n_1$, electromagnetic energy at optical frequencies propagates along the fiber waveguide through TIR at core-cladding interface.

d. Ray Optics Representation

- Simple ray optic representation is useful to understand the mechanism of wave propagation in an ideal step index optical waveguide.
- Basically **two kind of rays** propagate in a fiber, **meridional and skew rays**.
- **Meridional Rays** are confined to the meridional plane of the fibre.
- Meridional Rays lie in a single plane.

Cond...

- Meridional Rays are further classified into **2 categories**.
  a. **Bounded Rays**, are trapped in the core and propagate along the fiber axis.
  b. **Unbounded Rays** are refracted out of the fiber core.
- **Skew Rays** follow helical path along the fiber, they are not confined to a single plane.
- Skew rays are actually leaky rays. So greater power loss arises.

Cond…

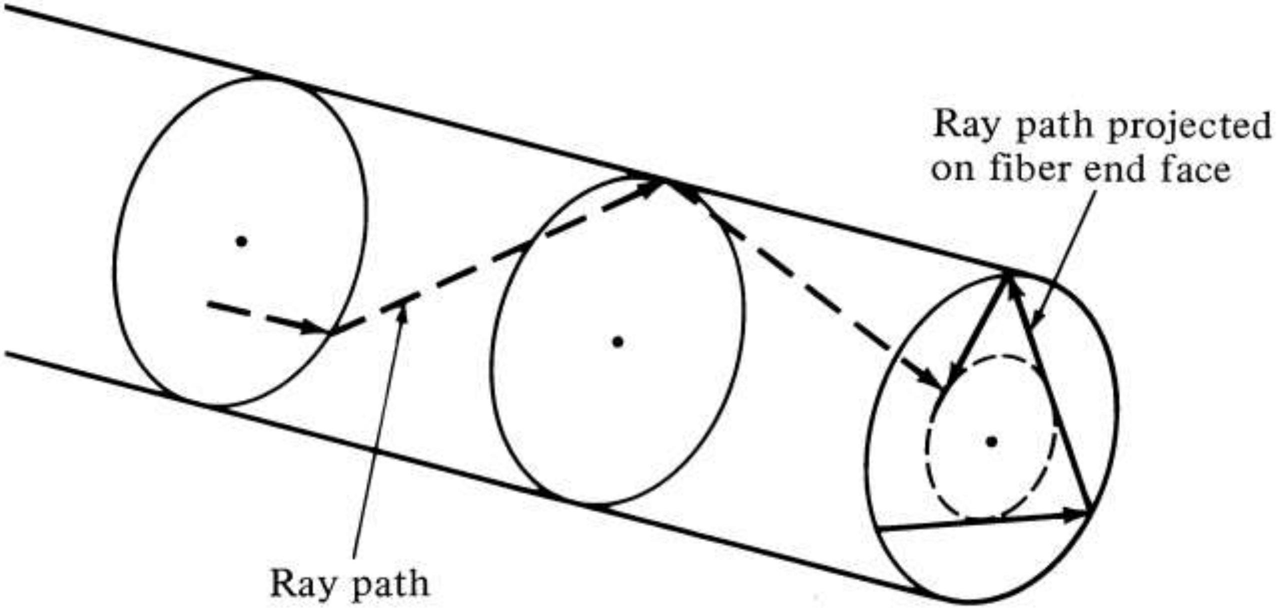


Fig.: Skew rays

Cond…

- The leaky rays are partially confined to the fiber core.
- They suffer attenuation as the light travels along the optical waveguide.
- The meridional ray is shown on next slide.
- The light ray enters the fiber core from medium of refractive index n at an angle θ_0 with respect to the fiber axis.
- The light ray then strikes the core-cladding at a normal angle ϕ .

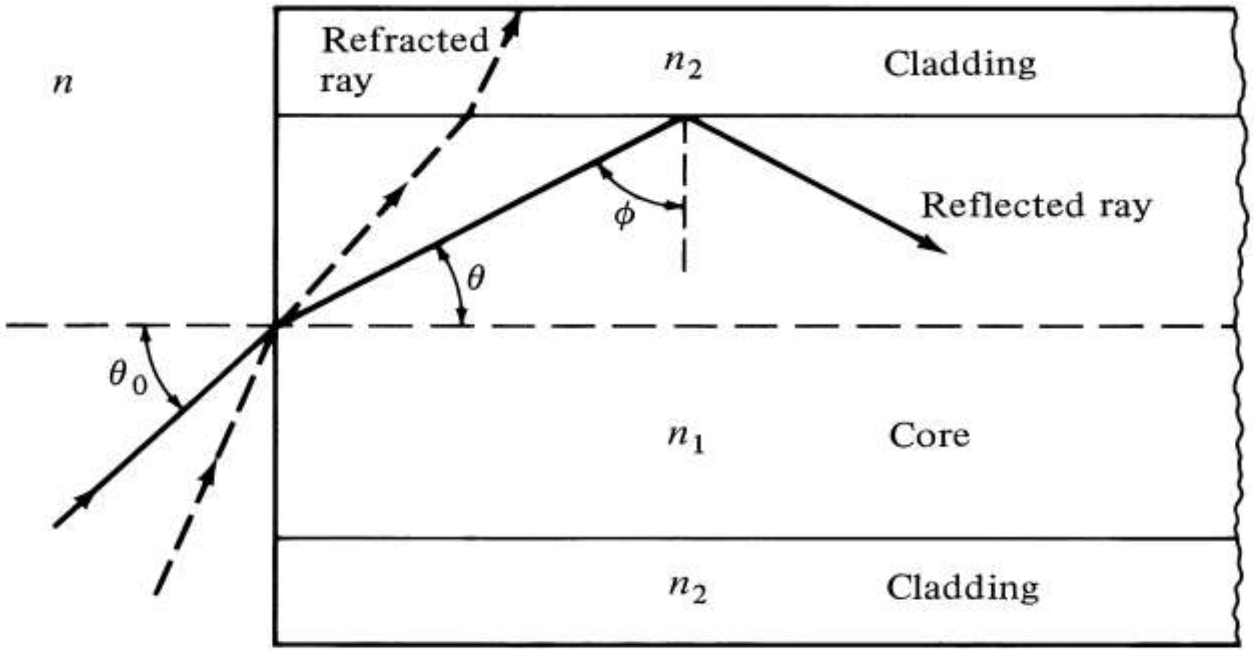


Fig: Meridional ray representation

Cond...

- It the ray follows TIR, then meridional ray follows a zigzag path along the fiber core.
- In this case meridional rays passes through the axis of the waveguide.
- From Snell's law, the minimum angle ϕ that supports TIR for the meridional ray is given by $\sin\phi_{min} = \frac{n_2}{n_1}$

- If the ray strikes less than this angle, then it will refract out of the core and will be lost in cladding.

Cond...

- At air-fiber core boundary, the Snell's law is related to maximum entrance angle as,

$$n\sin\theta_{0,\max} = n_1\sin\theta_c = \sqrt{n_1^2 - n_2^2}$$

- Where, $\phi_c = \frac{\pi}{2} - \theta_c$
- The above equation defines NA of a step-index fiber.

$$NA = n\sin\theta_{0,\max} = (n_1{}^2 - n_2{}^2)^{1/2} \approx n_1\sqrt{2\Delta}$$

- NA is used to describe light acceptance of a fiber.
- It is dimensionless quantity.

Problems

- Consider a multimode silica fiber that has a core refractive index 1.48 and a cladding refractive index 1.46. Find Critical angle, NA and acceptance angle in air for the fiber. [80.5^0, 0.242, 14^0]

- A silica optical fiber with a core diameter large enough to be considered by ray theory has a core refractive index of 1.50 and a cladding refractive index of 1.47. Determine critical angle, NA and acceptance angle in air for the fiber [78.5^0, 0.30, 17.4^0]

Attenuation

- The signal attenuation within the optical fibers, is usually expressed in logarithmic unit of the decibel.
- The decibel is usually used to compare two power levels.
- It is defined as the ratio of input power (transmitted)to the output power (received).

Cont...

- No of decibels (dB)= $10\log_{10}\frac{P_i}{P_0}$
- In optical communications the attenuation is usually expressed in decibels per unit length (i.e., $dBkm^{-1}$).
- Where α_{dB} is the signal attenuation per unit length in decibels and L is the fiber length.

$$\alpha_{dB} = -\frac{10}{L}\log_{10}\frac{P_0}{Pi}$$

Cont...

- According to the Beer's Law; $\frac{dP}{dz} = -\alpha P$
- This is the rate of change of optical power with respect to the path covered by the light ray.
- If we are going to launch the input power Pin from a fiber of length L, the out power will be given by:

$$P_0 = P_{in} e^{-\alpha L}$$

- So the attenuation coefficient can be expressed in dB/km as:

$$\alpha_{\mathrm{dB/km}} = -\frac{10}{L} \log_{10} \frac{P_O}{Pi}$$

Problems

- Ex. 3.1, senior
- Prob. 3.1, 3.2, 3.3, senior
- Ex. 3.1, 3.2, keiser
- Prob. 3.2, 3.3 ,3.4, keiser

Attenuation factors [1]

- Absorption
- Scattering losses
- Bending losses
- Core and cladding losses

Absorption

- Absorption in optical fiber is caused by these three mechanisms.
  1. Absorption by atomic defects in the glass composition
  2. Extrinsic absorption by impurity atoms in the glass material
  3. Intrinsic absorption by the basic constituent atoms of the fiber material.

Cont…

- Atomic defects means that the atoms are not on their proper place in crystal lattice, they are displaced.
- If the atoms are missing or there is high density of atoms in crystal lattice or there is oxygen in the glass, it is said to be atomic defects.

Cont…

- Material absorption can be divided into two categories.
- **Intrinsic absorption** losses correspond to absorption by fused silica (material used to make fibers)
- whereas **extrinsic absorption** is related to losses caused by impurities within silica.

Cont…

- Extrinsic absorption results from the presence of impurities.
- Metal impurities such as Fe, Cu, Co, Ni, Mn, and Cr absorb strongly in the wavelength range 0.6–1.6 μm.
- Their amount should be reduced to below 1 part per billion to obtain a loss level below 1dB/km.

Cont…

- The main source of extrinsic absorption is the presence of water vapors in the silica fibers.
- So in order to reduce these kind of losses dry fiber is used.
- The OH ion concentration is reduced in dry fibers.
- Such fibers can be used to transmit WDM signals.

Scattering losses

- Scattering means the transfer of some or all of the optical power contained within one propagating mode into a different mode.
- The power may be transferred to a leaky mode which means that the propagation is not within the core but it is within the fiber.
- Since the glass is made up of several oxides, SiO_2, P_2O_5, so compositional fluctuations may occur.

Cont...

- These may vary refractive index of the glass because of the different material used for the fabrication.
- This refractive index variation may cause Rayleigh scattering of the light.
- This occurs mainly because of the variation in wavelength.
- The scattering losses basically depend upon random molecular nature and various oxide constituent of the glass.

Cont…

- For a single component glass the scattering loss at a particular wavelength from density fluctuations is given by:

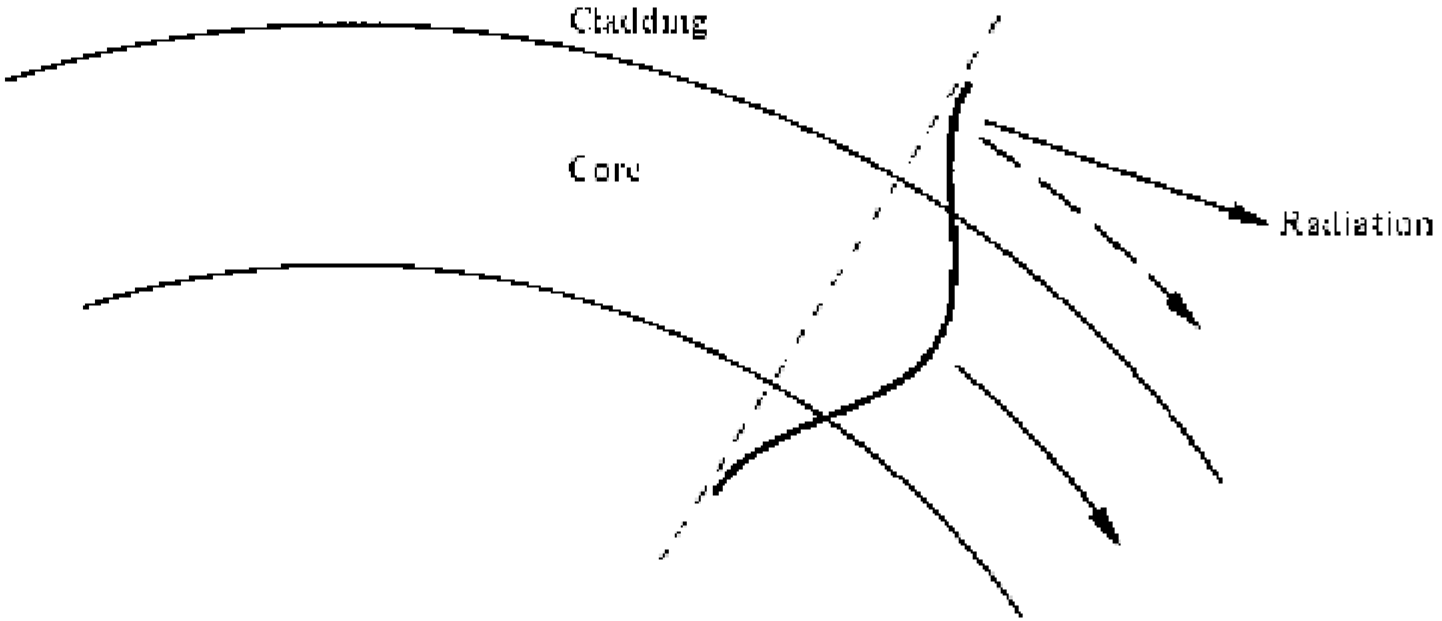
$$\alpha_{scat} = \frac{8\pi^3}{3\lambda^4} n^8 p^2 k_B T_f \beta_T$$

β_T *is isothermal compressibility of the material*

p *is the photo elastic coefficient*

Bending Loss

- Optical fibers suffer radiation losses at bends or curves on their paths.
- The bending losses are due to the energy in the existing field at bend exceeds the velocity of light in the cladding.
- The part of mode in the cladding needs to move faster than the velocity of light in that medium in order to keep the mode field in the fiber core.


Cladding
Core
Radiation

Cond...

- But this is not possible, so the energy associated with this part will be radiated out of the cladding.
- The bending loss can be represented by radiation attenuation coefficient. $\alpha_r = c_1 e^{-(c_2 R)}$
- Where R is the radius of curvature of the fiber bend & c_1 & c_2 are the constant.
- R_c is the critical curvature radius at which large bending loss occurs. $Rc \simeq \dfrac{3n_1^2\lambda}{4\pi(n_1^2 - n_2^2)^{3/2}}$

Cond...

- From the expression we can observe that the bending loss may be reduced by:
  I. Designing fiber with large relative refractive index differences.
  II. Operating at shortest possible wavelength.

Example

Two step index fibers have the following characteristics:

(a) A core refractive index of 1.500 with a relative refractive index difference of 0.2% and an operating wavelength of 1.55 μm.

(b) A core refractive index the same as (a) but a relative refractive index difference of 3% and an operating wavelength of 0.82 μm.

Estimate the critical radius of curvature at which large bending losses occur in both cases.

Core and Cladding Losses

- Core and cladding of any optical fiber have different refractive indices and the composition.
- So they have different attenuation coefficients α_{core} & α_{clad}.

Cond...

- If the mode coupling is ignored, the loss for a mode of order (v, m) for a step-index fiber is given by:

- The total loss of the waveguide can be evaluated by summing loss for all modes.

$$\alpha_{vm} = \alpha_{core}\frac{P_{core}}{P} + \alpha_{clad}\frac{P_{clad}}{P} = \alpha_{core} + (\alpha_{clad} - \alpha_{core})\frac{P_{clad}}{P}$$

Pulse broadening in optical fiber

- In digital communication systems, information to be sent is first coded in the form of pulses.
- These pulses of light are then transmitted from the transmitter to the receiver.
- At receiver the information is decoded.

Cond...

- The larger the number of pulses that can be sent per unit time at the receiver end, the larger will be the transmission capacity of the system.
- A pulse of light sent into a fiber broadens in time as it propagates through the fiber.
- This phenomenon is known as pulse dispersion.

Cond...

- Basically 2 kind of dispersions.
  I. Intramodal Dispersion
     a. Material Dispersion
     b. Waveguide Dispersion
  II. Intermodal Dispersion
- If the pulse dispersion occurs in single mode fiber, it is said to be ***intramodal dispersion***.
- It takes place when group velocity becomes a function of wavelength.

Cond...

- The ***group velocity*** is the speed at which the energy of the particular mode travels along the fiber.
- Since the spectral width is the band of wavelength over which the source emits light.
- So the intramodal dispersion increases with increase in spectral width.

Cond...

- Intramodal dispersion can be further classified into 2 categories.
  a. Material Dispersion
  b. Waveguide Dispersion

a) ***Material Dispersion:*** when the refractive index of the core varies with variation in the wavelength, Material Dispersion takes place. It is also called chromatic or spectral dispersion.

Cond…

b) ***Waveguide Dispersion:*** it occurs because the single mode fiber confines only 80% optical power to the core. The remaining 20% optical power travels in the clad faster than that in core. So the dispersion occurs.

Cond...

- ***Intermodal Dispersion:*** If each mode have different value of group velocity at a single frequency, Intermodal Dispersion takes place. It occurs in multimode fibers.

Material Dispersion

- Takes place when the index of the refraction varies as a function of the optical wavelength.
- Consider an electrical signal that modulates an optical source.
- Let this modulated optical excites all modes equally at the input of the fiber.

Cond...

- So each mode will carry equal amount of optical energy through the fiber.
- Each mode contains all the spectral components in the wavelength band over which the source emits light.
- So each spectral component will travel independently in the fiber.

Cond…

- Each component will undergo a time delay or group delay per unit length in the direction of propagation & is given by:

$$\frac{\tau_g}{L} = \frac{1}{c}\frac{d\beta}{dk} = -\frac{\lambda^2}{2\pi c}\frac{d\beta}{d\lambda} \left\{ because\, k = \frac{2\pi}{\lambda} \rightarrow so\ dk = -\frac{2\pi}{\lambda^2} d\lambda \right\}$$

$$L \rightarrow the\, dis\tan ce\, traveled\, by\, a\ pulse$$

$$\beta = kn \rightarrow the\ propagation\, constant\, along\, the\ fiber\, axis$$

$$k = 2\pi / \lambda \rightarrow wave\ propagation\, constant$$

$$V_g = \frac{L}{\tau_g} = \left(\frac{1}{c}\frac{d\beta}{dk}\right)^{-1} = c\frac{dk}{d\beta} = -\frac{2\pi c}{\lambda^2}\frac{d\lambda}{d\beta}$$

Cond...

- V_g is called the group velocity at which the energy in a pulse travels along a fiber.
- Material dispersion occurs because the index of refraction varies as a function of the optical wavelength.
- Since the group velocity V_g of a mode is a function of the index of refraction.

Cond…

- The various spectral components of a given mode will travel at different speed, depending on the wavelength.
- So, the material dispersion is therefore an Intramodal dispersion.

Cond...

- It occurs in single-mode fiber and for LED because LED has a broader output spectrum than LASER.
- The propagation constant is given by:

$$\beta = kn(\lambda) = \frac{2\pi n(\lambda)}{\lambda}$$

cond...

$$\Rightarrow \frac{d\beta}{dk} = \frac{d}{dk}\left(\frac{2\pi n(\lambda)}{\lambda}\right) = 2\pi \frac{d}{dk}\left(\frac{n(\lambda)}{\lambda}\right) = 2\pi \frac{d}{d(2\pi/\lambda)}\left(\frac{n(\lambda)}{\lambda}\right)$$

$$\Rightarrow \frac{d}{d(1/\lambda)}\left(\frac{n(\lambda)}{\lambda}\right) = \frac{d}{d\lambda^{-1}}\left(\lambda^{-1} n(\lambda)\right) = \lambda^{-1}\frac{dn(\lambda)}{d\lambda^{-1}} + n(\lambda)\frac{d\lambda^{-1}}{d\lambda^{-1}}$$

$$\Rightarrow \lambda^{-1}\frac{dn(\lambda)}{d\lambda^{-1}} + n(\lambda) = n(\lambda) + \lambda^{-1}\frac{dn(\lambda)}{d\lambda^{-1}} = n(\lambda) - \lambda\frac{dn(\lambda)}{d\lambda}$$

- So, the group delay resulting from material dispersion is given by:

$$\tau_{mat} = \frac{L}{c}\frac{d\beta}{dk} = \frac{L}{c}\left(n - \lambda\frac{dn}{d\lambda}\right)$$

Cond...

- The pulse spreading due to material dispersion is given by:

$$\sigma_{mat} = \frac{d}{d\lambda}\left[\frac{L}{c}\left(n-\lambda\frac{dn}{d\lambda}\right)\right]\sigma_\lambda = L\sigma_\lambda\frac{1}{c}\left[\frac{dn}{d\lambda}-\lambda\frac{d^2n}{d\lambda^2}-\frac{dn}{d\lambda}\right]$$

$$= -L\sigma_\lambda\frac{\lambda}{c}\frac{d^2n}{d\lambda^2} = \left|D_{mat}(\lambda)\right|L\sigma_\lambda$$

$$\Rightarrow \left|D_{mat}(\lambda)\right| = \frac{\lambda}{c}\left|\frac{d^2n}{d\lambda^2}\right|$$

- Where, $D_{mat}(\lambda)$ is the material dispersion & σ_λ is the spectral width of the source .

Problems:

- Example 3.6 Senior
- Problem 3.14 Senior

Waveguide Dispersion

- In this case, the pulse spreading doesn't depends upon the wavelength of the material.
- It is because the refractive index is independent of the wavelength of the material.
- it results from the variation in the group velocity in the wavelength for a particular mode.
- We know that the group delay is the time required for a mode to travel along a fiber, let the it's length is L.

Cond…

- For small vales of the index difference the *normalized propagation constant b* can be approximated by:

$$b = \left(\frac{wa}{V}\right)^2 = 1 - \left(\frac{ua}{V}\right)^2 = \frac{\beta^2 / k^2 - n_2^2}{n_1^2 - n_2^2}$$

$$b \approx \frac{\beta / k - n_2}{n_1 - n_2}$$

$$\beta \approx n_2 k (b\Delta + 1)$$

cond...

- The group delay due to waveguide dispersion is given by:

$$\tau_{wg} = \frac{L}{c}\frac{d\beta}{dk} = \frac{L}{c}\left[n_2 + n_2\Delta\frac{d(kb)}{dk}\right]$$

- So the group delay can be given by in terms of V as:

$$\tau_{wg} = \frac{L}{c}\left[n_2 + n_2\Delta\frac{d(Vb)}{dV}\right]$$

$$Because \qquad V = ka\left(n_1^2 - n_2^2\right)^{1/2} \approx kan_1\sqrt{2\Delta}$$

cond...

- the pulse spreading due to waveguide dispersion is given by:

$$\sigma_{wg} = \left|\frac{d\tau_{wg}}{d\lambda}\right|\sigma_\lambda = \left|D_{wg}(\lambda)\right| L\sigma_\lambda = \frac{V}{\lambda}\left|\frac{d\tau_{wg}}{d\lambda}\right|\sigma_\lambda = \frac{n_2 L\Delta\sigma_\lambda}{c\lambda} V\frac{d^2(Vb)}{dV^2}$$

$$\left|D_{wg}(\lambda)\right| = \frac{n_2\Delta}{c\lambda} V\frac{d^2(Vb)}{dV^2} \rightarrow Waveguide\ Dispersion$$

Problems:

- Example 3.8, Keiser

Pulse Broadening in Graded index fiber Waveguides [3]

- Section 3.3, keiser 2^{nd} Edition.

Mode theory for Circular waveguide [1]

a. Overview of modes
b. Summary of key modal concepts
c. Maxwell’s Equations
d. Waveguide Equations Wave Equations for Step Index Fibers
e. Modal Equation
f. Modes in Step-Index fiber
g. Linearly polarized modes

Overview of modes

- The optical waveguide is the fundamental element that interconnects the various devices of an optical integrated circuit.
- Optical waves travel in the waveguide in distinct optical modes.
- A mode, in this sense, is a spatial distribution of optical energy in one or more dimensions that remains constant in time.

Cond…

- The mode theory uses electromagnetic wave behavior to describe the propagation of light along a fiber.
- A set of guided electromagnetic waves is called the **modes** of the fiber.
- For a given mode, a change in wavelength can prevent the mode from propagating along the fiber.

Cond...

- If the mode is no longer bound to the fiber, this mode is said to be cut off.
- The wavelength at which a mode is cutoff is called the **cutoff wavelength** for that mode.
- However, an optical fiber is always able to propagate at least one mode.
- This mode is referred to as the fundamental mode of the fiber.
- The fundamental mode can never be cut off.

Cond…

- The wavelength that prevents the next higher mode from propagating is called the **cutoff wavelength** of the fiber.
- An optical fiber that operates above the cutoff wavelength (at a longer wavelength) is called a **single mode fiber**.
- An optical fiber that operates below the cutoff wavelength is called a **multimode fiber**.

Cond...

- In a fiber, the propagation constant of a plane wave is a function of the wave's wavelength and mode.
- Maxwell's equations describe electromagnetic waves or modes as having two components.
- The two components are the electric field, $E(x, y, z)$, and the magnetic field, $H(x, y, z)$.
- The electric field, E, and the magnetic field, H, are at right angles to each other.

Cond...

- Modes traveling in an optical fiber are said to be transverse.
- The transverse modes, shown in Fig. 2.5, propagate along the axis of the fiber.
- In TE modes, the electric field is perpendicular to the direction of propagation.
- The magnetic field is in the direction of propagation.

Cond...

- Another type of transverse mode is the transverse magnetic (TM) mode.
- TM modes are opposite to TE modes.
- In TM modes, the magnetic field is perpendicular to the direction of propagation.
- The electric field is in the direction of propagation.
- Fig. 2.5 shows only TE modes.

Cond...

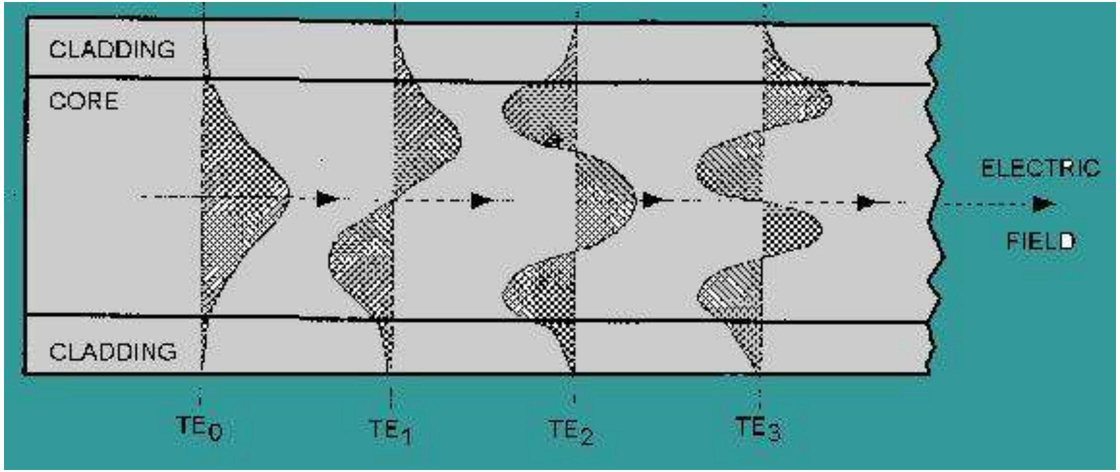


Fig. 2.5 Electric field distribution for several of lower - order guided modes

Cond…

- The TE mode field patterns shown in Fig. 2.5 indicate the **order** of each mode.
- The order of each mode is indicated by the number of field maxima within the core of the fiber.
- For example, TE_0 has one field maxima.
- The electric field is maximum at the center of the waveguide and decays toward the core-cladding boundary.

Cond...

- TE_0 is considered the fundamental mode or the lowest order standing wave.
- As the number of field maxima increases, the order of the mode is higher.
- Generally, modes with more than a few (5-10) field maxima are referred to as high-order modes.
- The order of the mode is also determined by the angle the wave-front makes with the axis of the fiber.

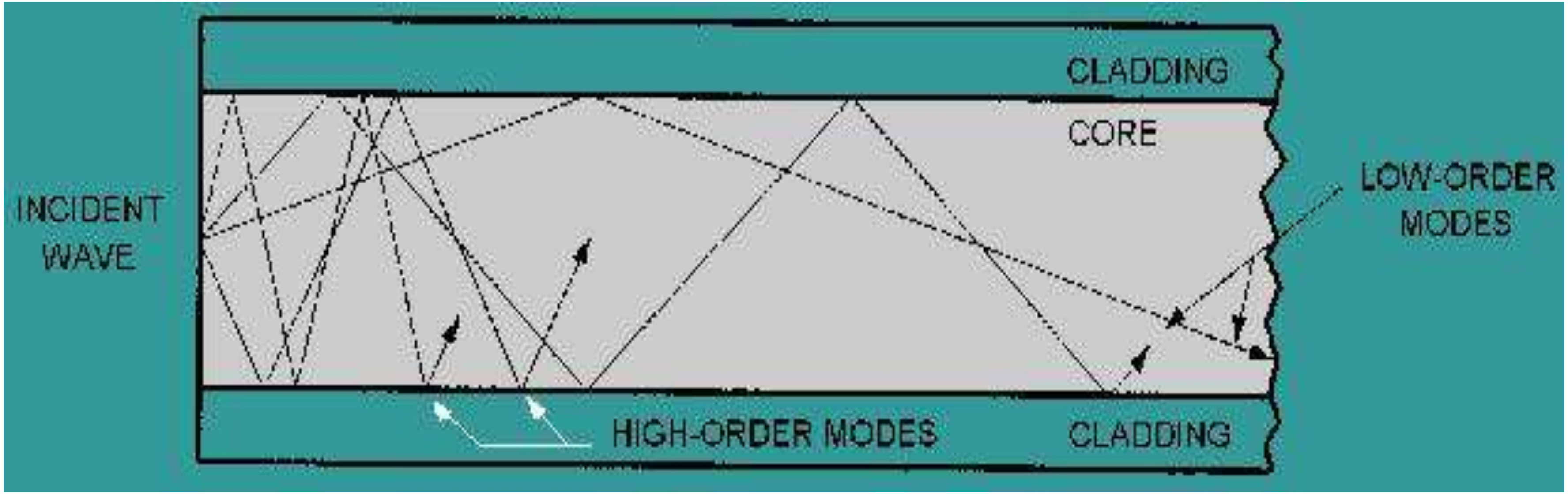


Fig. 2.6 Low-order and high-order modes

Cond...

- Fig. 2.6 illustrates light rays as they travel down the fiber.
- These light rays indicate the direction of the wave-fronts.
- High-order modes cross the axis of the fiber at steeper angles.
- Low-order and high-order modes are shown in Fig. 2.6.
- Low-order modes penetrate the cladding only slightly.

Cond...

- In low-order modes, the electric and magnetic fields are concentrated near the center of the fiber.
- In high-order modes, the electrical and magnetic fields are distributed more toward the outer edges of the fiber.
- The penetration of low-order and high-order modes into the cladding region indicates that some portion is refracted out of the core.

Cond...

- As the core and the cladding modes travel along the fiber, mode coupling occurs.
- **Mode coupling** is the exchange of power between two modes.
- Mode coupling to the cladding results in the loss of power from the core modes.

Maxwell's Equations

- To analyze the optical waveguide we need to consider Maxwell's equations that give the relationships between the electric and magnetic fields.
- There are 4 Maxwell's Equations,

$$\nabla \; X \; E = -\frac{\partial B}{\partial t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

$$\nabla \; X \; H = \frac{\partial D}{\partial t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1b)$$

$$\nabla . D = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1c)$$

$$\nabla . B = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1d)$$

- where D = εE and B = μH. The parameter ε is the permittivity (or dielectric constant) and μ is the permeability of the medium.
- Taking the curl of Eq. (1a) and use Eq. (1b) gives,

$$\nabla X\,(\nabla X\,E) = -\mu\frac{\partial}{\partial t}(\nabla X\,H) = -\varepsilon\mu\frac{\partial^2 E}{\partial t^2} \qquad (2a)$$

- Using the vector identity

$$\nabla X\,(\nabla X\,E) = \nabla(\nabla.E) - \nabla^2 E$$

- And using Eq. (1c), Eq. (2a) becomes

$$\nabla^2 E = \varepsilon\mu\frac{\partial^2 E}{\partial t^2} \qquad (2b)$$

- Similarly, by taking the curl of Eq. (1b), it can be shown that

$$\nabla^2 H = \varepsilon\mu \frac{\partial^2 H}{\partial t^2} \qquad (2c)$$

- Equations (2b) and (2c) are the standard *wave equations.*

$$v_p = \sqrt{\frac{1}{\varepsilon\mu}}$$

$$c = \sqrt{\frac{1}{\varepsilon_0\mu_0}}$$

Waveguide Equations

- If the electromagnetic waves are to propagate along the z axis, they will have a functional dependence of the form

$$E = E_0(r,\phi)e^{j(\omega t-\beta z)} \quad (3a)$$

$$H = H_0(r,\phi)e^{j(\omega t-\beta z)} \quad (3b)$$

- The parameter β is the z component of the propagation vector and will be determined by the boundary conditions on the electromagnetic fields at the core – cladding interface.

$$\nabla \; X \; E = \hat{r}\left(\frac{1}{r}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{1}{r}\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r}\right) + \hat{z}\,\frac{1}{r}\left(\frac{\partial (rE_\phi)}{\partial r} - \frac{\partial E_r}{\partial \phi}\right) = -\frac{\partial B}{\partial t}$$

$$\Rightarrow \frac{1}{r}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -\frac{\partial B_r}{\partial t} = -\mu\frac{\partial H_r}{\partial t} = -\mu(j\omega)H_r$$

$$\Rightarrow \frac{1}{r}\frac{\partial E_z}{\partial \phi} - (-j\beta)E_\phi = -j\omega\mu H_r$$

$$\Rightarrow \frac{1}{r}\frac{\partial E_z}{\partial \phi} + j\beta E_\phi = -j\omega\mu H_r$$

$$\Rightarrow \frac{1}{r}\left[\frac{\partial E_z}{\partial \phi} + jr\beta E_\phi\right] = -j\omega\mu H_r$$

- When Eq. (3a) and (3b) are substituted into Maxwell's curl equations, we have

$$\frac{1}{r}\left[\frac{\partial E_z}{\partial \phi} + jr\beta E_\phi\right] = -j\omega\mu H_r \qquad (4a)$$

$$j\beta E_r + \frac{\partial E_z}{\partial r} = j\omega\mu H_\phi \qquad (4b)$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rE_\phi\right) - \frac{\partial E_r}{\partial \phi}\right] = -j\omega\mu H_z \qquad (4c)$$

- And, from Eq. (1b),

$$\frac{1}{r}\left[\frac{\partial H_z}{\partial \phi} + jr\beta H_\phi\right] = j\omega\varepsilon E_r \qquad (5a)$$

$$j\beta H_r + \frac{\partial H_z}{\partial r} = -j\omega\varepsilon E_\phi \qquad (5b)$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rH_\phi\right) - \frac{\partial H_r}{\partial \phi}\right] = j\omega\varepsilon E_z \qquad (5c)$$

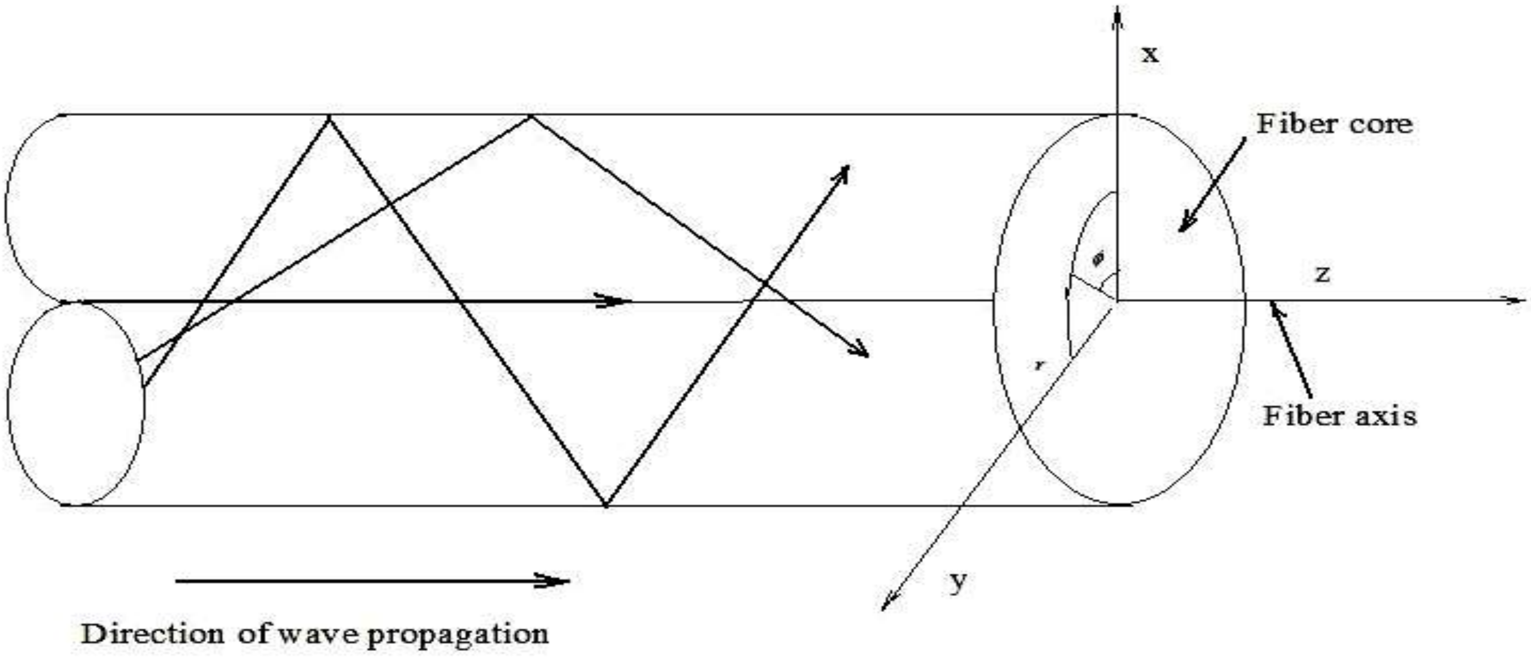


Fig. 2 Cylindrical coordinate system used for analyzing electromagnetic wave propagation in an optical fiber

- when E_z and H_z are known, the remaining transverse components E_r, E_ϕ, H_r and H_ϕ can be determined.
- H_ϕ or E_r can be found in terms of E_z or H_z by eliminating E_ϕ or H_r from Eq. (4a) and Eq. (5b). These yields,

$$E_r = -\frac{j}{q^2}\left\{\beta\frac{\partial E_z}{\partial r} + \frac{\mu\omega}{r}\frac{\partial H_z}{\partial \phi}\right\} \qquad (6a)$$

$$E_\phi = -\frac{j}{q^2}\left\{\frac{\beta}{r}\frac{\partial E_z}{\partial \phi} - \mu\omega\frac{\partial H_z}{\partial r}\right\} \qquad (6b)$$

$$H_r = -\frac{j}{q^2}\left\{\beta\frac{\partial H_z}{\partial r} - \frac{\omega\varepsilon}{r}\frac{\partial E_z}{\partial \phi}\right\} \qquad (6c)$$

$$H_\phi = -\frac{j}{q^2}\left\{\frac{\beta}{r}\frac{\partial H_z}{\partial \phi} + \omega\varepsilon\frac{\partial E_z}{\partial r}\right\} \qquad (6d)$$

Where $q^2 = \omega^2\mu\varepsilon - \beta^2 = k^2 - \beta^2$

- Substitution of Eq. (6c) and Eq. (6d) into Eq. (5c) gives the wave equation in cylindrical coordinates,

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + q^2 E_z = 0 \qquad (7a)$$

- And substitution of Eq. (6a) and Eq. (6b) into Eq. (4c) gives,

$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r}\frac{\partial H_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 H_z}{\partial \phi^2} + \frac{\partial^2 H_z}{\partial z^2} + q^2 H_z = 0 \qquad (7b)$$

- Mode solutions can be obtained in which either $E_z = 0$ or $H_z = 0$.
- When $E_z = 0$ the modes are called *transverse electric* or TE modes, and when $H_z = 0$ they are called *transverse magnetic* or TM modes.
- *Hybrid* mode exists if both H_z and E_z are nonzero. These are designated as HE or EH a mode, depending on whether Hz and Ez, respectively, can makes a larger contribution to the transverse field.

Wave Equations for Step – Index Fibers

- We can use Eq. (7a) and Eq. (7b) to find the guided modes in a step – index fiber.
- A standard mathematical procedure for solving equations such as Eq. (7a) is to use the separation – of – variables method, which assumes a solution of the form

$$E_z = AF_1(\mathrm{r})F_2(\phi)F_3(\mathrm{z})F_4(\mathrm{t}) \qquad (8a)$$

- Assume the time- and z- dependent factors are given by

$$F_3(\mathrm{z})F_4(\mathrm{t}) = \mathrm{e}^{j(\omega t-\beta z)} \qquad (8b)$$

- We assume a periodic function of the form

$$F_2(\phi) = e^{j\nu\phi} \qquad (8c)$$

- Thus constant ν can be positive or negative, but it must be an integer since the field must be periodic in ϕ with a period of 2π.
- Now substituting Eq. (8c) and Eq. (8b) into Eq. (8a), the wave equation for E_z [Eq. (7a)] becomes

$$\frac{\partial^2 F_1}{\partial r^2} + \frac{1}{r}\frac{\partial F_1}{\partial r} + \left(q^2 - \frac{\nu^2}{r^2}\right)F_1 = 0 \qquad (8d)$$

- This is well - known differential equation for Bessel functions.

Derivation for Eq. 8d

- When we substitute Eq. (8c) and Eq. (8b) into Eq. (8a), the wave equation for E_z [Eq. (7a)] becomes

$$E_z = AF_1(\mathrm{r})F_2(\phi)F_3(\mathrm{z})F_4(\mathrm{t})$$

$$E_z = AF_1(\mathrm{r})e^{j\nu\phi}\mathrm{e}^{j(\omega t-\beta z)}$$

$$E_z = AF_1(\mathrm{r})e^{j(\nu\phi+\omega t-\beta z)}$$

Since we know that:

$$\frac{\partial^2 E_z}{\partial r^2}+\frac{1}{r}\frac{\partial E_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 E_z}{\partial \phi^2}+\frac{\partial^2 E_z}{\partial z^2}+q^2 E_z=0$$

- Put E_z in this equation, we will get

Cond...

- So,

$$\frac{\partial^2 (AF_1(\mathrm{r})e^{j(\nu\phi+\omega t-\beta z)})}{\partial r^2}+\frac{1}{r}\frac{\partial (AF_1(\mathrm{r})e^{j(\nu\phi+\omega t-\beta z)})}{\partial r}$$

$$+\frac{1}{r^2}\frac{\partial^2 (AF_1(\mathrm{r})e^{j(\nu\phi+\omega t-\beta z)})}{\partial \phi^2}+\frac{\partial^2 (AF_1(\mathrm{r})e^{j(\nu\phi+\omega t-\beta z)})}{\partial z^2}$$

$$+q^2 AF_1(\mathrm{r})e^{j(\nu\phi+\omega t-\beta z)}=0$$

$$\Rightarrow Ae^{j(\nu\phi+\omega t-\beta z)}\frac{\partial^2 F_1(\mathrm{r})}{\partial r^2}+\frac{1}{r}Ae^{j(\nu\phi+\omega t-\beta z)}\frac{\partial F_1(\mathrm{r})}{\partial r}$$

$$+\frac{1}{r^2}Ae^{j(\nu\phi+\omega t-\beta z)}(j\nu)^2 F_1(\mathrm{r})+Ae^{j(\nu\phi+\omega t-\beta z)}(-j\beta)^2 F_1(\mathrm{r})$$

$$+q^2 AF_1(\mathrm{r})e^{j(\nu\phi+\omega t-\beta z)}=0$$

Cond...

$$\Rightarrow Ae^{j(\nu\phi+\omega t-\beta z)}\left[\frac{\partial^2 F_1(\mathrm{r})}{\partial r^2}+\frac{1}{r}\frac{\partial F_1(\mathrm{r})}{\partial r}+\frac{1}{r^2}(j\nu)^2 F_1(\mathrm{r})+(-j\beta)^2 F_1(\mathrm{r})+q^2 F_1(\mathrm{r})\right]=0$$

$$\Rightarrow \frac{\partial^2 F_1(\mathrm{r})}{\partial r^2}+\frac{1}{r}\frac{\partial F_1(\mathrm{r})}{\partial r}-\frac{\nu^2}{r^2}F_1(\mathrm{r})-\beta^2 F_1(\mathrm{r})+q^2 F_1(\mathrm{r})=0$$

$$\Rightarrow \frac{\partial^2 F_1(\mathrm{r})}{\partial r^2}+\frac{1}{r}\frac{\partial F_1(\mathrm{r})}{\partial r}+\left(q^2-\frac{\nu^2}{r^2}-\beta^2\right)F_1(\mathrm{r})=0$$

$$\Rightarrow \frac{\partial^2 F_1}{\partial r^2}+\frac{1}{r}\frac{\partial F_1}{\partial r}+\left(q^2-\frac{\nu^2}{r^2}\right)F_1=0$$

- Three different types of Bessel functions

If q **is real** then the solutions are

Bessel functions of **first kind** $J_\nu(qr)$

Bessel functions of **second kind** $Y_\nu(qr)$

If q **is imaginary,** then the solutions are

Modified Bessel functions of **first kind** $I_\nu(qr/j)$

Modified Bessel functions of **second kind** $K_\nu(qr/j)$

If q **is complex,** then the solutions are

Hankel functions of **first kind** $H_\nu^{(1)}(qr)$

Hankel functions of **second kind** $H_\nu^{(2)}(qr)$

- The quantity ν is called the **order** of the function and (qr) is called the **argument** of the Bessel function.

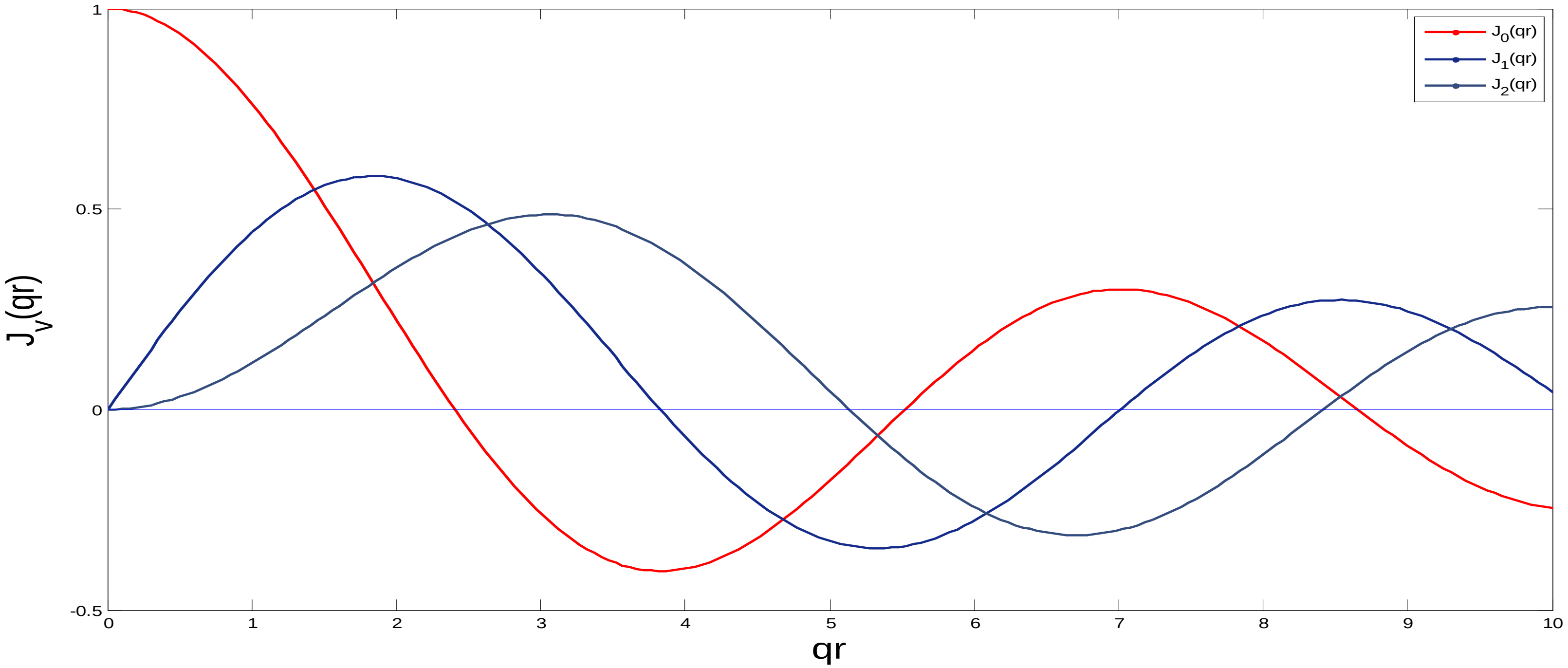


Fig. 3 Bessel functions of first kind

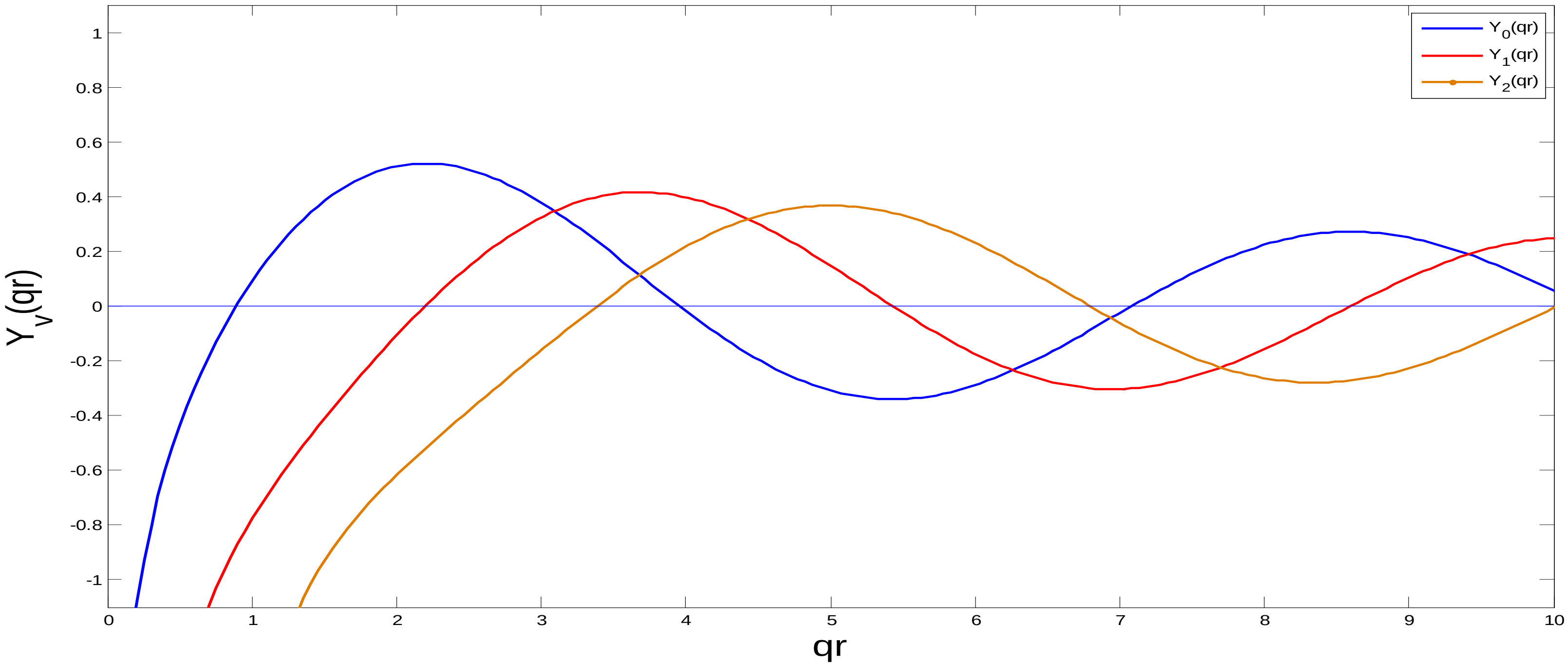


Fig. 4 Bessel functions of second kind

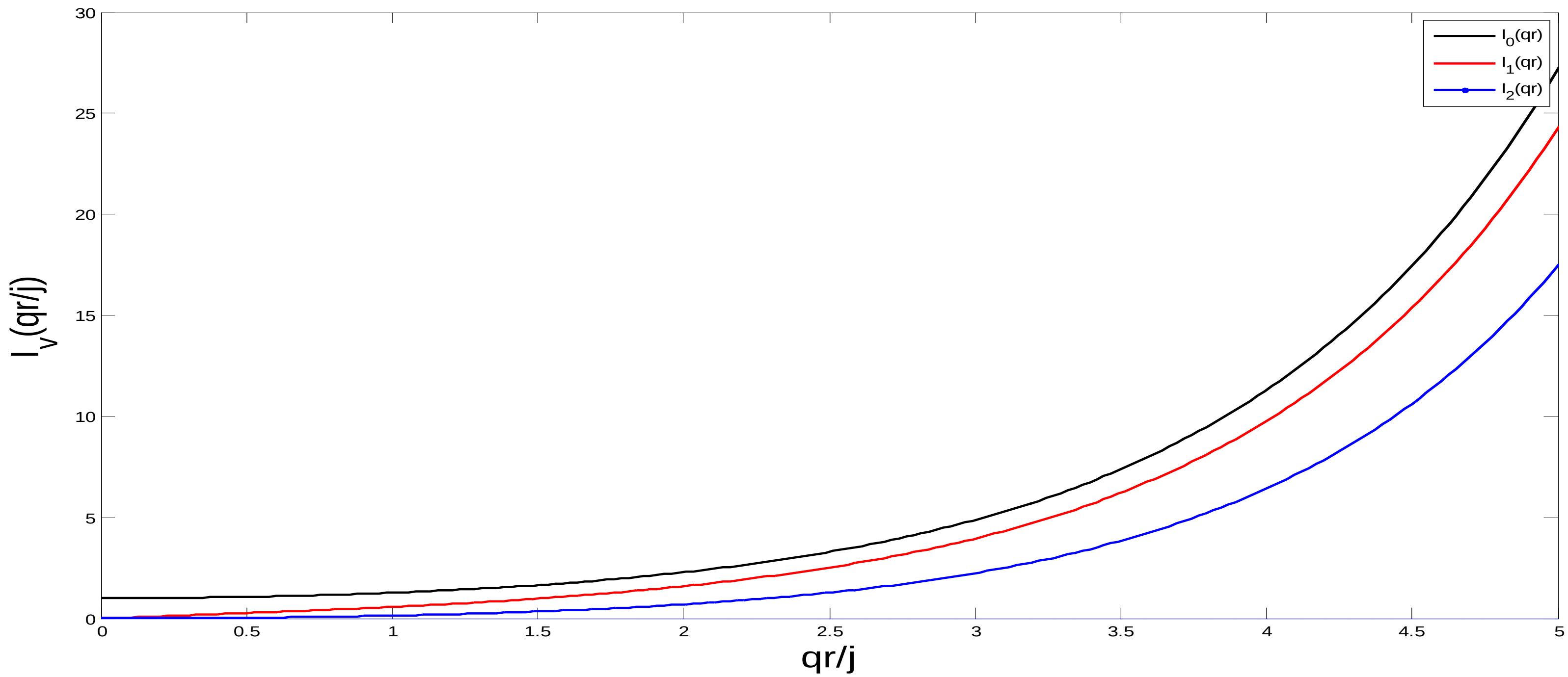


Fig. 5 Modified Bessel functions of first

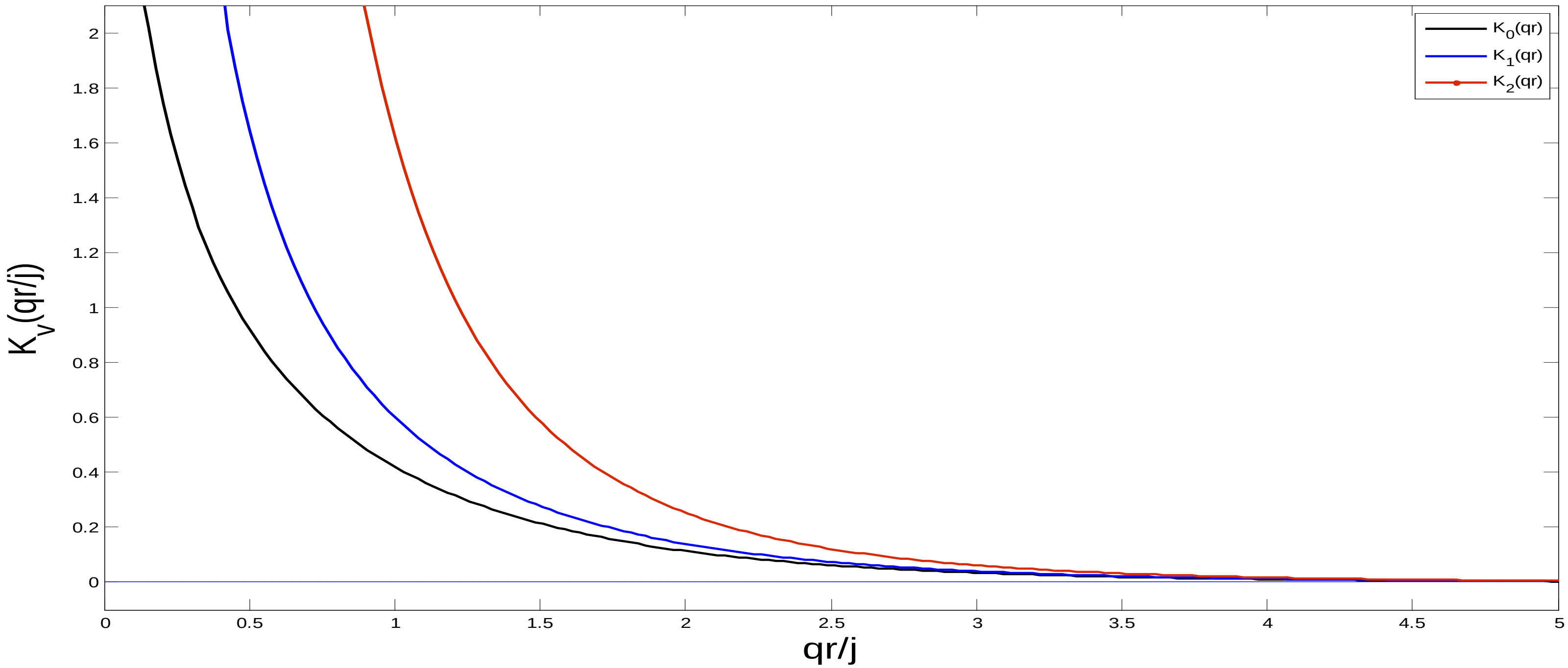


Fig. 6 Modified Bessel functions of second kind

- **Bessel function of first kind:** The functions are finite for all values of *r*.
- **Bessel function of second kind:** The functions start from $-\infty$ at $r = 0$ and have finite values for all the other values of *r*.
- **Modified Bessel function of first kind:** The functions increases monotonically with increases of r.
- **Modified Bessel function of second kind:** The functions decreases monotonically with increases of r.

Therefore we can conclude that

- Bessel function of **1st** kind $J_\nu(ur)$ is the appropriate solution for the modal fields inside the core of an optical fiber.

  where, $u^2 = k_1^2 - \beta^2$ with $k_1 = \frac{2\pi n_1}{\lambda}$

- **Modified** Bessel function of **2nd** kind $K_\nu(wr)$ is the appropriate solution for the modal fields outside the core of an optical fiber.

  where, $w^2 = \beta^2 - k_2^2$ with $k_2 = \frac{2\pi n_2}{\lambda}$

- The expressions for E_z and H_z inside the core are, when (r < a)

$$E_{z1} = AJ_\nu(ua)e^{j\nu\phi - j\beta z + j\omega t} \qquad (9a)$$

$$H_{z1} = BJ_\nu(ua)e^{j\nu\phi - j\beta z + j\omega t} \qquad (9b)$$

- The expressions for E_z and H_z outside the core are, when (r > a)

$$E_{z2} = CK_\nu(wa)e^{j\nu\phi - j\beta z + j\omega t} \qquad (9c)$$

$$H_{z2} = DK_\nu(wa)e^{j\nu\phi - j\beta z + j\omega t} \qquad (9d)$$

- where ***A***, ***B***, ***C*** & ***D*** are arbitrary constants which are to be evaluated from the boundary conditions.

- For a guided mode, the propagation constant lies between two limits k_1 and k_2.
- If $n_2 k = k_2 \leq \beta \leq k_1 = n_1 k$ then a field distribution is generated which will has an oscillatory behavior in the core and a decaying behavior in the cladding.
- The energy then is propagated along fiber without any loss.

- Where $k = \frac{2\pi}{\lambda}$ is free – space propagation constant.

Modal Equation

- At inner core – cladding boundary ($E = E_{z1}$) and at the outside of the boundary ($E = E_{z2}$), so

$$E_{z1} - E_{z2} = AJ_\nu(ua) - CK_\nu(wa) = 0 \quad \text{.....(1.1)}$$

$$E_{\phi1} - E_{\phi2} = -\frac{j}{u^2}\left[A\frac{j\nu\beta}{a}J_\nu(ua) - B\omega\mu u J_\nu'(ua)\right] - \frac{j}{w^2}\left[C\frac{j\nu\beta}{a}K_\nu(wa) - D\omega\mu w K_\nu'(wa)\right] = 0 \quad \text{.....(1.2)}$$

- Similarly, for **H**,

$$H_{z1} - H_{z2} = BJ_\nu(ua) - DK_\nu(wa) = 0 \quad \text{.....(1.3)}$$

$$H_{\phi1} - H_{\phi2} = -\frac{j}{u^2}\left[B\frac{j\nu\beta}{a}J_\nu(ua) + A\omega\varepsilon_1 u J_\nu'(ua)\right] - \frac{j}{w^2}\left[D\frac{j\nu\beta}{a}K_\nu(wa) + C\omega\varepsilon_1 w K_\nu'(wa)\right] = 0 \quad \text{.....(1.4)}$$

Cond...

- Above equations are set of four equations with four unknown coefficients, A, B, C and D.
- A solution to these equations exists only if the determinant of these coefficients is zero, that is

$$\begin{vmatrix} A1 & B1 & C1 & D1 \\ A2 & B2 & C2 & D2 \\ A3 & B3 & C3 & D3 \\ A4 & B4 & C4 & D4 \end{vmatrix} = 0$$

- $A1$ to $A4$, $B1$ to $B4$, $C1$ to $C4$, & $D1$ to $D4$ are coefficients of A in Eq. (1.1), Eq. (1.2), Eq. (1.3) and Eq. (1.4).

$$A1 = J_\nu(ua) \qquad B1 = 0 \qquad C1 = -K_\nu(wa) \qquad D1 = 0$$

$$A2 = \frac{\beta\nu}{au^2} J_\nu(ua) \qquad B2 = \frac{j\omega\mu}{u} J_\nu{}'(ua) \qquad C2 = \frac{\beta\nu}{aw^2} K_\nu(wa) \qquad D2 = \frac{j\omega\mu}{w} K_\nu{}'(wa)$$

$$A3 = 0 \qquad B3 = J_\nu(ua) \qquad C3 = 0 \qquad D3 = -K_\nu(wa)$$

$$A4 = -\frac{j\omega\varepsilon_1}{u} J_\nu{}'(ua) \qquad B4 = \frac{\beta\nu}{au^2} J_\nu(ua) \qquad C4 = -\frac{j\omega\varepsilon_2}{w} K_\nu{}'(wa) \qquad D4 = \frac{\beta\nu}{aw^2} K_\nu(wa)$$

$$\begin{vmatrix} J_\nu(ua) & 0 & -K_\nu(wa) & 0 \\ \frac{\beta\nu}{au^2} J_\nu(ua) & \frac{j\omega\mu}{u} J_\nu{}'(ua) & \frac{\beta\nu}{aw^2} K_\nu(wa) & \frac{j\omega\mu}{w} K_\nu{}'(wa) \\ 0 & J_\nu(ua) & 0 & -K_\nu(wa) \\ -\frac{j\omega\varepsilon_1}{u} J_\nu{}'(ua) & \frac{\beta\nu}{au^2} J_\nu(ua) & -\frac{j\omega\varepsilon_2}{w} K_\nu{}'(wa) & \frac{\beta\nu}{aw^2} K_\nu(wa) \end{vmatrix} = 0$$

COND...

$$\Rightarrow \begin{vmatrix} A1 & 0 & C1 & 0 \\ A2 & B2 & C2 & D2 \\ 0 & B3 & 0 & D3 \\ A4 & B4 & C4 & D4 \end{vmatrix} = 0$$

$$\Rightarrow A1\begin{vmatrix} B2 & C2 & D2 \\ B3 & 0 & D3 \\ B4 & C4 & D4 \end{vmatrix} + C1\begin{vmatrix} A2 & B2 & D2 \\ 0 & B3 & D3 \\ A4 & B4 & D4 \end{vmatrix} = 0$$

$$\Rightarrow A1\left[B2\begin{vmatrix} 0 & D3 \\ C4 & D4 \end{vmatrix} - C2\begin{vmatrix} B3 & D3 \\ B4 & D4 \end{vmatrix} + D2\begin{vmatrix} B3 & 0 \\ B4 & C4 \end{vmatrix} \right] + C1\left[A2\begin{vmatrix} B3 & D3 \\ B4 & D4 \end{vmatrix} + B2\begin{vmatrix} 0 & D3 \\ A4 & D4 \end{vmatrix} + D2\begin{vmatrix} 0 & B3 \\ A4 & B4 \end{vmatrix} \right] = 0$$

$$\Rightarrow A1\left[B2(-C4D3) - C2(B3D4 - B4D3) + D2(B3C4)\right] + C1\left[A2(B3D4 - B4D3) + B2(A4D3) + D2(-A4B3)\right] = 0$$

$$\Rightarrow -A1B2C4D3 - A1B3C2D4 + A1B4C2D3 + A1B3C4D2 + A2B3C1D4 - A2B4C1D3 + A4B2C1D3 - A4B3C1D2 = 0$$

$$\Rightarrow -[J_\nu(ua)]\left[\frac{j\omega\mu}{u}J_\nu{}'(ua)\right]\left[-\frac{j\omega\varepsilon_2}{w}K_\nu{}'(wa)\right][-K_\nu(wa)]-[J_\nu(ua)][J_\nu(ua)]\left[\frac{\beta\nu}{aw^2}K_\nu(wa)\right]\left[\frac{\beta\nu}{aw^2}K_\nu(wa)\right]$$

$$+[J_\nu(ua)]\left[\frac{\beta\nu}{au^2}J_\nu(ua)\right]\left[\frac{\beta\nu}{aw^2}K_\nu(wa)\right][-K_\nu(wa)]+[J_\nu(ua)][J_\nu(ua)]\left[-\frac{j\omega\varepsilon_2}{w}K_\nu{}'(wa)\right]\left[\frac{j\omega\mu}{w}K_\nu{}'(wa)\right]$$

$$+\left[\frac{\beta\nu}{au^2}J_\nu(ua)\right][J_\nu(ua)][-K_\nu(wa)]\left[\frac{\beta\nu}{aw^2}K_\nu(wa)\right]-\left[\frac{\beta\nu}{au^2}J_\nu(ua)\right]\left[\frac{\beta\nu}{au^2}J_\nu(ua)\right][-K_\nu(wa)][-K_\nu(wa)]$$

$$+\left[-\frac{j\omega\varepsilon_1}{u}J_\nu{}'(ua)\right]\left[\frac{j\omega\mu}{u}J_\nu{}'(ua)\right][-K_\nu(wa)][-K_\nu(wa)]-\left[-\frac{j\omega\varepsilon_1}{u}J_\nu{}'(ua)\right][J_\nu(ua)][-K_\nu(wa)]\left[\frac{j\omega\mu}{w}K_\nu{}'(wa)\right]=0$$

$$\Rightarrow \frac{\omega^2\mu\varepsilon_2}{uw}J_\nu(ua)K_\nu(wa)J_\nu{}'(ua)K_\nu{}'(wa)-\frac{\beta^2\nu^2}{a^2w^4}J^2{}_\nu(ua)K^2{}_\nu(wa)$$

$$-\frac{\beta^2\nu^2}{a^2u^2w^2}J^2{}_\nu(ua)K^2{}_\nu(wa)+\frac{\omega^2\mu\varepsilon_2}{w^2}J^2{}_\nu(ua)K'^2{}_\nu(wa)$$

$$-\frac{\beta^2\nu^2}{a^2u^2w^2}J^2{}_\nu(ua)K^2{}_\nu(wa)-\frac{\beta^2\nu^2}{a^2u^4}J^2{}_\nu(ua)K^2{}_\nu(wa)$$

$$+\frac{\omega^2\mu\varepsilon_1}{u^2}J'^2{}_\nu(ua)K^2{}_\nu(wa)+\frac{\omega^2\mu\varepsilon_1}{uw}J_\nu(ua)K_\nu(wa)J_\nu{}'(ua)K_\nu{}'(wa)=0$$

$$\Rightarrow \frac{\omega^2\mu(\varepsilon_1+\varepsilon_2)}{uw} J_\nu(ua)K_\nu(wa)J_\nu{}'(ua)K_\nu{}'(wa) - \frac{\beta^2\nu^2}{a^2}\left(\frac{1}{u^4}+\frac{1}{w^4}\right) J^2{}_\nu(ua)K^2{}_\nu(wa)$$

$$-2\frac{\beta^2\nu^2}{a^2u^2w^2} J^2{}_\nu(ua)K^2{}_\nu(wa) + \frac{\omega^2\mu\varepsilon_1}{u^2} J'^2{}_\nu(ua)K^2{}_\nu(wa) + \frac{\omega^2\mu\varepsilon_2}{w^2} J^2{}_\nu(ua)K'^2{}_\nu(wa) = 0$$

$$\Rightarrow \frac{k_1{}^2+k_2{}^2}{uw} J_\nu(ua)K_\nu(wa)J_\nu{}'(ua)K_\nu{}'(wa) + \frac{\omega^2\mu\varepsilon_1}{u^2} J'^2{}_\nu(ua)K^2{}_\nu(wa) + \frac{\omega^2\mu\varepsilon_2}{w^2} J^2{}_\nu(ua)K'^2{}_\nu(wa)$$

$$= 2\frac{\beta^2\nu^2}{a^2u^2w^2} J^2{}_\nu(ua)K^2{}_\nu(wa) + \left(\frac{\beta\nu}{a}\right)^2\left(\frac{1}{u^4}+\frac{1}{w^4}\right) J^2{}_\nu(ua)K^2{}_\nu(wa)$$

$$\Rightarrow (k_1{}^2+k_2{}^2)\frac{J_\nu{}'(ua)}{uJ_\nu(ua)}\frac{K_\nu{}'(wa)}{wK_\nu(wa)} + \frac{k_1{}^2}{u^2} J'^2{}_\nu(ua)K^2{}_\nu(wa) + \frac{k_2}{w^2} J^2{}_\nu(ua)K'^2{}_\nu(wa) = 2\frac{\beta^2\nu^2}{a^2u^2w^2} + \left(\frac{\beta\nu}{a}\right)^2\left(\frac{1}{u^4}+\frac{1}{w^4}\right)$$

$$\Rightarrow (k_1{}^2+k_2{}^2)\frac{J_\nu{}'(ua)}{uJ_\nu(ua)}\frac{K_\nu{}'(wa)}{wK_\nu(wa)} + \frac{k_1{}^2}{u^2} J'^2{}_\nu(ua)K^2{}_\nu(wa) + \frac{k_2}{w^2} J^2{}_\nu(ua)K'^2{}_\nu(wa) = 2\left(\frac{\beta\nu}{a}\right)^2\frac{1}{u^2}\frac{1}{w^2} + \left(\frac{\beta\nu}{a}\right)^2\left(\frac{1}{u^4}+\frac{1}{w^4}\right)$$

$$\Rightarrow (k_1^{\,2}+k_2^{\,2})\frac{J_\nu'(ua)}{uJ_\nu(ua)}\frac{K_\nu'(wa)}{wK_\nu(wa)}+\frac{k_1^{\,2}}{u^2}J'^2_\nu(ua)K^2_\nu(wa)+\frac{k_2}{w^2}J^2_\nu(ua)K'^2_\nu(wa)=\left(\frac{\beta\nu}{a}\right)^2\left(\frac{1}{u^4}+\frac{1}{w^4}+2\frac{1}{u^2}\frac{1}{w^2}\right)$$

$$\Rightarrow k_1^{\,2}\frac{J_\nu'(ua)}{uJ_\nu(ua)}\frac{K_\nu'(wa)}{wK_\nu(wa)}+k_2^{\,2}\frac{J_\nu'(ua)}{uJ_\nu(ua)}\frac{K_\nu'(wa)}{wK_\nu(wa)}+\frac{k_1^{\,2}}{u^2}J'^2_\nu(ua)K^2_\nu(wa)+\frac{k_2}{w^2}J^2_\nu(ua)K'^2_\nu(wa)=\left(\frac{\beta\nu}{a}\right)^2\left(\frac{1}{u^2}+\frac{1}{w^2}\right)^2$$

$$\Rightarrow k_1^{\,2}\left\{\frac{J_\nu'(ua)}{uJ_\nu(ua)}\frac{K_\nu'(wa)}{wK_\nu(wa)}+\frac{1}{u^2}J'^2_\nu(ua)K^2_\nu(wa)\right\}+k_2^{\,2}\left\{\frac{J_\nu'(ua)}{uJ_\nu(ua)}\frac{K_\nu'(wa)}{wK_\nu(wa)}+\frac{1}{w^2}J^2_\nu(ua)K'^2_\nu(wa)\right\}=\left(\frac{\beta\nu}{a}\right)^2\left(\frac{1}{u^2}+\frac{1}{w^2}\right)^2$$

$$\Rightarrow \left\{\frac{J_\nu'(ua)}{uJ_\nu(ua)}+\frac{K_\nu'(wa)}{wK_\nu(wa)}\right\}\left\{k_1^2\frac{J_\nu'(ua)}{uJ_\nu(ua)}+k_2^2\frac{K_\nu'(wa)}{wK_\nu(wa)}\right\}=\left(\frac{\beta\nu}{a}\right)^2\left(\frac{1}{u^2}+\frac{1}{w^2}\right)^2$$

Cond…

- Evaluation of the above determinant yields the following eigenvalue equation for β.

$$\left\{\frac{J_\nu'(ua)}{uJ_\nu(ua)}+\frac{K_\nu'(wa)}{wK_\nu(wa)}\right\}\left\{k_1^2\frac{J_\nu'(ua)}{uJ_\nu(ua)}+k_2^2\frac{K_\nu'(wa)}{wK_\nu(wa)}\right\}=\left(\frac{\beta\nu}{a}\right)^2\left(\frac{1}{u^2}+\frac{1}{w^2}\right)^2$$

- This equation is called characteristic equation.

Modes in Step Index Fiber

- The *cutoff condition* is the point at which a mode is no longer bound to the core region.
- So its field no longer decays on the outside of the core. The cutoffs for the various modes can be found by solving characteristic equation in the limit $w^2 \rightarrow 0$.

Table Cutoff conditions for some lower – order modes

ν	**Mode**	**Cutoff condition**
0	TE_{0m}, TM_{0m}	$J_0(ua) = 0$
1	HE_{1m}, EH_{1m}	$J_1(ua) = 0$
$\geq$ **2**	$EH_{\nu m}$	$J_\nu(ua) = 0$
	$HE_{\nu m}$	$\left(\frac{n_1^2}{n_2^2} + 1\right) J_{\nu-1}(ua) = \frac{ua}{\nu - 1} J_\nu(ua)$

Cond...

- The permissible range of β for bound solutions is therefore

$$n_2 k = k_2 \le \beta \le k_1 = n_1 k$$

- An important parameter connected with the cutoff condition is the *normalized frequency* V (also called the V *number* or V *parameter*) defined by

$$V^2 = (u^2 + w^2)a^2 = \left(\frac{2\pi a}{\lambda}\right)^2 (n_1{}^2 - n_2{}^2) = \left(\frac{2\pi a}{\lambda}\right)^2 NA^2$$

Cond...

- Which is dimensionless number that determines how many modes a fiber can support.
- The number of modes that can exist in a wave guide as a function of V may be conveniently represented in terms of a *normalized propagation constant* b defined by

$$b = \left(\frac{aw}{V}\right)^2 = \left(\frac{(\beta / k)^2 - n_2^2}{n_1^2 - n_2^2}\right)$$

- A plot of b as function of V is shown in **Fig.** for few of the lower – order modes.

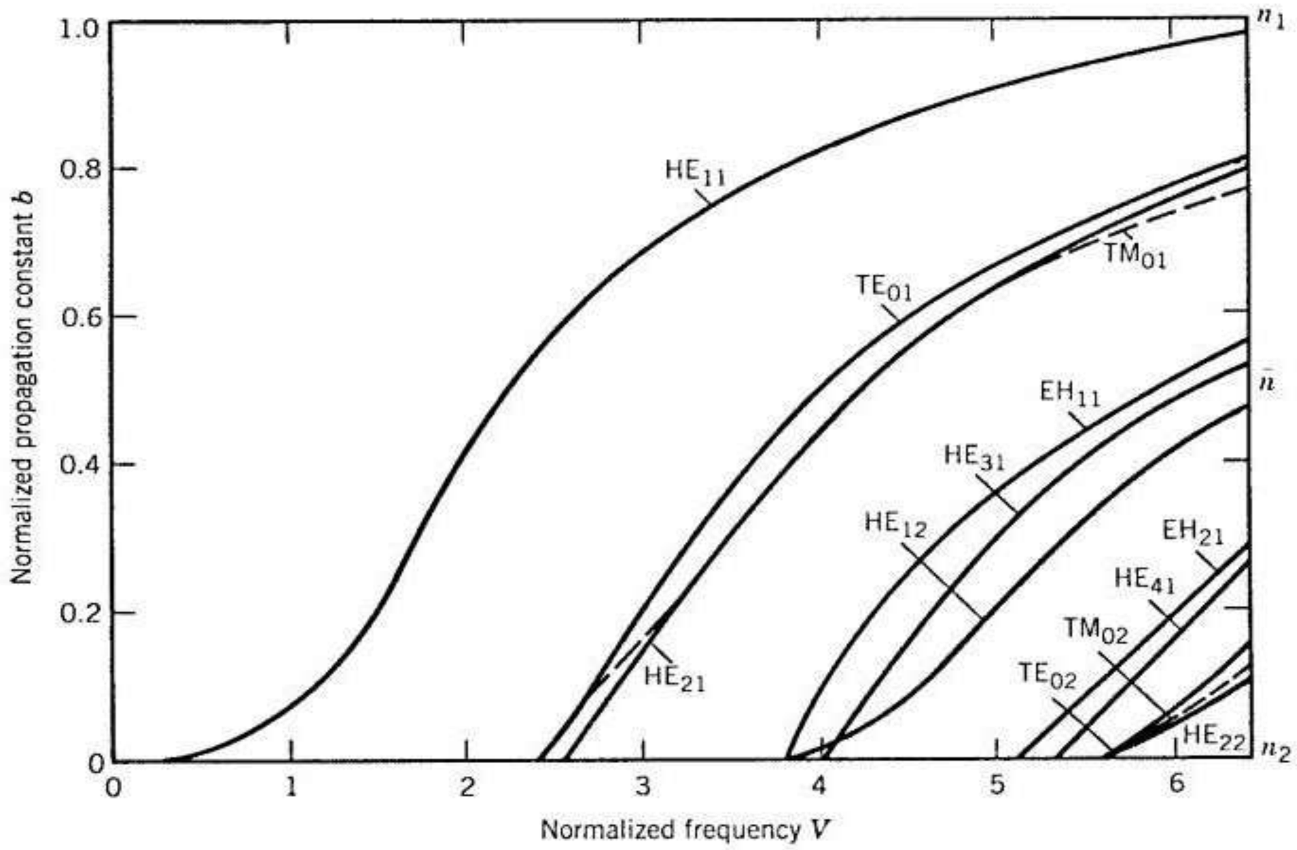


Fig. Plots of the propagation constant *b* as a function of *V* for a lower – order modes

Cond...

- This figure shows that each mode can exist only for values of V that exceed a certain limiting value.
- The modes are cutoff when $\beta/k = n_2$. The HE_{11} mode has no cutoff.
- This is the principle on which the single mode fiber is based. By appropriately choosing a, n_1 and n_2 so that

Cond...

$$V = \left(\frac{2\pi a}{\lambda}\right)(n_1^{\,2} - n_2^{\,2})^{1/2} \leq 2.405$$

- Which is the value at which the lowest – order Bessel function $J_0 = 0$, all modes except the HE_{11} mode are cutoff.

Summary of key modal concepts

- V number is an important parameter that is connected with the cutoff condition & is defined by:

$$V^2 = (u^2 + w^2)a^2 = \left(\frac{2\pi a}{\lambda}\right)^2 ({n_1}^2 - {n_2}^2) = \left(\frac{2\pi a}{\lambda}\right)^2 NA^2$$

$$V = \left(\frac{2\pi a}{\lambda}\right) NA$$

Cond...

- It is a dimensionless number.
- It determines how many modes a fiber can support.
- It also used to express the number of modes M in a multimode fiber when V is large.

$$M = \frac{1}{2}\left(\frac{2\pi a}{\lambda}\right)^2 (n_1^{\,2} - n_2^{\,2}) = \frac{V^2}{2}$$

Problem

- Example 2.4, Senior

Single Mode Fiber

- The single mode fiber is advantageous as compared to multi mode fiber.
- Since it propagates only a single mode so there is no possibility of signal dispersion due different modes.
- It has small core cladding difference.
- For single mode fiber, V is always smaller than 2.405.

Mode Field Diameter

- MFD is the fundamental parameter for a single mode fiber.
- It can be determined from mode field distribution of lower order guided mode, i.e., in this case fundamental mode.
- MFD is analogous to the core diameter of multi mode fiber.
- If the mode field distribution is Gaussian,

$$E(r) = E_0 e^{-\left(\frac{r}{W_0}\right)^2}$$

- E_0, is the field at the centre of the fiber, W_0, width of the electric field distribution.

Cond...

- The MFD width is given by:

$$2W_0 = 2\left[\frac{2\int_0^\infty r^3E^2(r)dr}{\int_0^\infty rE^2(r)dr}\right]1/2 = E_0 e^{-\left(\frac{r}{W_0}\right)^2}$$

- Where E(r) is the mode field distribution.

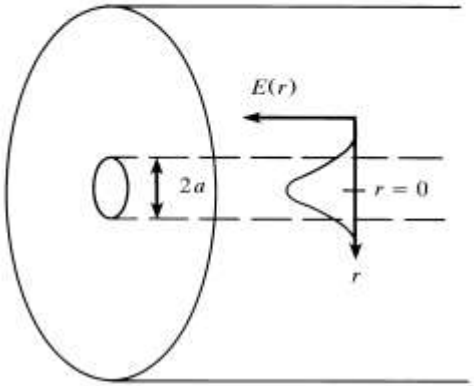
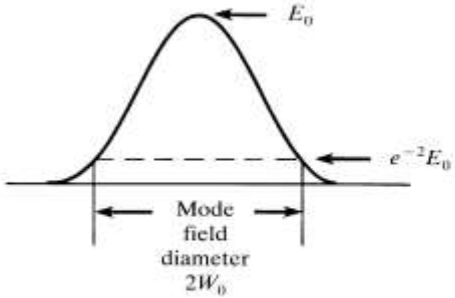


Fig.: Mode-field diameter



Mode Coupling

- The propagation characteristics of the optical fiber may be changed due to variation in the core diameter, irregularities at the core – cladding interference & refractive index variations.
- These are the factors responsible for the coupling of energy from one mode to another.

- From the figures we can observe that the ray doesn't maintains the same angle with the fiber axis.
- So, the individual mode is not able to propagate without the energy transfer to the adjacent mode.
- This is called **Mode Coupling**.

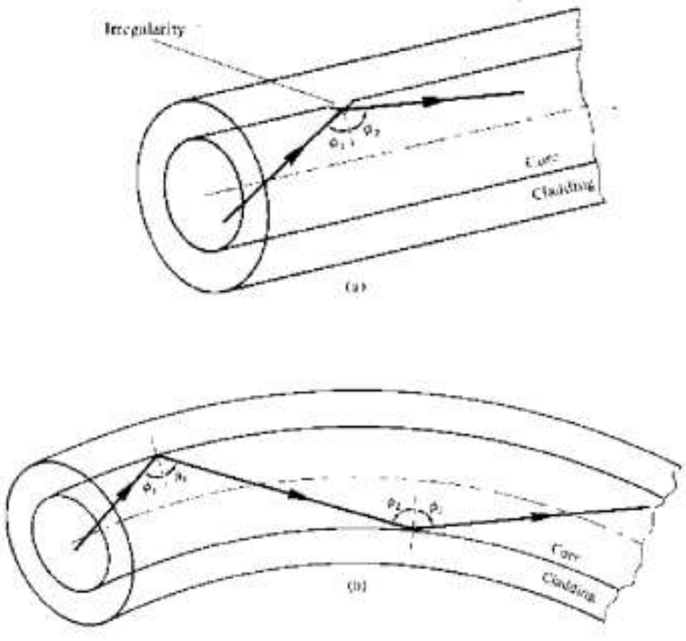


Fig: Possibilities for Mode Coupling

Problem

- Example 2.4, 2.8, Senior.

Review Problems

- Determine the refractive indices of the core and cladding material of a fiber if numerical aperture is 0.22 and refractive index difference is 0.012. **[UPTU 2009-10]**
- Find the maximum diameter allowed for a fiber having core refractive index 1.53 and cladding refractive index 1.50. the fiber is supporting only one mode of a wavelength of 1200 nm. **[UPTU 2009-10]**
- Explain the wave theory for optical propagation in a cylindrical waveguide. **[UPTU 2009-10]**
- Find the maximum diameter of a core for a single mode fiber operating at 1.55 μm with $n_1 = 1.55$ and $n_2 = 1.48$. **[UPTU 2009-10]**

Cond...

- Explain block diagram of a Optical fiber communication system. **[UPTU 2009-10]**
- Briefly explain the pulse broadening due to material dispersion in optical fiber. **[UPTU 2009-10]**
- Explain overall fiber dispersion in single mode fiber. **[UPTU 2009-10]**
- Explain linear and non linear scattering losses in optical fiber. **[UPTU 2009-10]**
- Differentiate between skew rays and meridional rays. Explain the nature of light. **[UPTU 2010-11]**
- Deduce the condition for total internal reflection of light in a fiber. **[UPTU 2010-11]**

Cond…

- Calculate the numerical aperture of a step index fiber having n_1 = 1.48 and n_2 = 1.46. What is the maximum entrance angle for this fiber if the outer medium is air with n = 1?**[UPTU 2010-11]**
- Write the advantages of optical communication. Also explain the fiber structure. **[UPTU 2010-11]**
- Write the Numerical Aperture profile for step index and graded index fiber. **[UPTU 2010-11]**
- What is attenuation? How it limits the performance of optical communication? Discuss the mechanism by which attenuation caused in optical signal propagation along fiber. **[UPTU 2010-11]**

Cond...

- What is signal distortion? How it limits the performance of optical communication? Discuss and derive the expression for mechanisms by which distortion is caused in optical fiber communication. **[UPTU 2010-11]**
- Explain pulse broadening in optical fibers. Discuss modal dispersion present in optical fiber. Calculate dispersion parameter. **[UPTU 2010-11]**
- Explain in brief the propagation characteristics of single and multimode fibers. **[UPTU 2011-12]**
- Explain various dispersion mechanisms **[UPTU 2011-12]**

Cond...

- What do you understand by scattering loss? Describe its types with expressions.**[UPTU 2011-12]**
- Differentiate between meridonal and skew rays. An optical fiber in air has NA 0.4; compare the acceptance angle for skew rays which changes direction by 100^0 at each reflection. **[UPTU 2011-12]**
- A graded index fiber has a core with parabolic refractive index profile and diameter 40 μm. Numerical aperture is 0.2. Estimate the total number of guided modes for a wavelength of 1 μm. **[UPTU 2011-12]**
- Explain the following: **[UPTU 2011-12]**
  - Normalized propagation constant
  - Mode field diameter.

Cond…

- What do you mean by Acceptance angle of an optical fiber? Show how it is related to refractive index of the fiber core, cladding and medium where fiber is placed. **[UPTU 2011-12]**
- Draw the block diagram of optical fiber communication system. Enlist the advantages of optical communication. **[UPTU 2011-12]**
- A continuous 12 km long optical fiber link has a loss of 1.5 dB/km. **[UPTU 2011-12]**
  - What is the minimum optical power level that must be launched into the fiber to maintain an optical power level of 0.3 μW at the receiving end?
  - What is the required input power if the fiber has loss of 2.5 dB/km?
- Discuss the propagation of light in the optical fiber using ray theory and determine the value of acceptance angle and numerical aperture of step index fiber. **[UTU 2011-12]**

Cond...

- Draw and explain the block diagram of optical fiber communication system. **[UTU 2011-12]**
- A multimode step index fiber is operating at wavelength of 850 nm with a core diameter of 80 μm and relative index difference of 1.5%. The refractive index of the core is 1.5. Determine the normalized frequency and the number of guided modes. **[UTU 2011-12]**
- Define V parameter of an optical fiber and calculate for a cylindrical optical fiber of radius 50 μm operating at a wavelength of 1500 nm having refractive indices 1.53 and 1.50 for core and cladding respectively. **[UTU 2011-12]**

Cond...

- Explain the concept of electromagnetic modes in relation to a planar optical waveguide. Discuss the modifications that may be made to electromagnetic mode theory in a planar waveguide in order to describe optical propagation in a cylindrical fiber. **[UTU 2011-12]**
- Describe with the help of simple ray diagrams. **[UTU 2011-12]**
  - The multimode step index fiber.
  - The single mode step index fiber.
- Compare the advantages and disadvantages of those fibers for use as an optical channel.
- Discuss absorption losses in optical fiber, comparing the intrinsic and extrinsic mechanisms. **[UTU 2011-12]**
- Discuss Dispersion mechanism with respect to single mode fiber indicating the dominating effects and then show how inter-modal dispersion may be minimize within the single mode region. **[UTU 2011-12]**

Cond...

- Briefly discuss with the aid of a suitable diagram what is meant by the acceptance angle for an optical fiber. Show how this is related to the fiber numerical aperture and the refractive indexes for the fiber core and cladding? **[UTU 2012-13]**
- A step index with a large core diameter compared with the wavelength of the transmitted light has an acceptance angle in air of 22 degree and a relative refractive index difference of 3%. Estimate the numerical aperture and the critical angle at the core-cladding interface of the fiber. **[UTU 2012-13]**
- What are the modes in optical fiber? Calculate number of modes for a graded index optical fiber if core diameter (d = 62.5 $\mu\, m$), NA =0.275 and Operating wavelength = 1300 nm. **[UTU 2012-13]**
- List the three major cause of attenuation in an optical fiber and explain their mechanism. **[UTU 2012-13]**

REFRENCES

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